Lesson 9.2: Solving Logs/Exponents

<u>Solving Logs:</u>
1. Isolate Log
2. Condense (max 1 log per side)
3. Rewrite to exponent
4. Solve
5. Check for extraneous solutions

(No negatives in the argument)

Argument $\neq 0$.

<u>Solving Exponents</u>
1. Isolate the exponent
2. Rewrite to a Log
3. Solve for x.

$$2\log x + 3 = 5$$

$$-3 \quad -3$$

$$2\log x = 2$$

$$\cancel{10}^{\log_{10} x^2} = 10^2$$

$$x^2 = 10^2$$

$$\sqrt{x^2} = \sqrt{100}$$

$$x = \pm 10$$

$$\boxed{x = 10}$$

-10 is extraneous

$$\log_4(x+3) + \log_4 x = \log_4 10$$

Solving Logs:

1. Isolate Log ✓
2. Condense (max 1 log per side) ✓
3. Rewrite to exponent ✓
4. Solve ✓
5. Check for extraneous solutions

$$4^{\log_4(x^2+3x)} = 4^{\log_4 10}$$

$$x^2 + 3x = 10$$

$$x^2 + 3x - 10 = 0$$

$$(x+5)(x-2) = 0$$

$x = -5$ extraneous

$\boxed{x = 2}$ ✓

$\log_4(-5+2) + \log_4(-5)$

Negative arguments

$$\ln(x+1)-2=3$$

$$+2 \quad +2$$

$$\cancel{e^{\ln}}(x+1)=e^{5}$$

$$x+1=e^{5}$$

$$-1 \quad -1$$

$$\boxed{x=e^{5}-1}$$

Solving Logs:

1. Isolate Log ✓
2. Condense (max 1 log per side) ✓
3. Rewrite to exponent ✓
4. Solve ✓
5. Check for extraneous solutions ✓

log x ⇒ base 10

ln x ⇒ base e!

$$\log_2(x+1) + \log_2 3 = 3$$

$$\log_2(3x+3) = 3$$

$$3x+3 = 2^3$$

$$3x+3 = 8$$

$$-3 \quad -3$$

$$\frac{3x}{3} = \frac{5}{3}$$

$$\boxed{x = 5/3}$$

$$\underline{\log_2\left(5/3+1\right)}$$

Positive

Keep the solution

$\boxed{2^{x+2}} - 4 = 5$

$+4 \quad +4$

$2^{x+2} = 9$

$\log_2(2^{x+2}) = \log_2(9)$

$x+2 = \log_2(9)$

$-2 \quad -2$

$\boxed{x = \log_2(9) - 2}$

<u>Solving Exponents</u>

1. Isolate the exponent ✓
2. Rewrite to a Log ✓
3. Solve for x.

① $\dfrac{2e^{x-1}}{2} = \dfrac{8}{2}$

$\log_e x = \ln x$

<u>Solving Exponents</u>

1. Isolate the exponent ✓
2. Rewrite to a Log ✓
3. Solve for x. ✓

$\ln e^{x-1} = \ln 4$

$x - 1 = \ln(4)$

$+1 \qquad +1$

$\boxed{x = \ln(4) + 1}$

$$10^{x-1} = 13$$

Solving Exponents

1. Isolate the exponent ✓
2. Rewrite to a Log ✓
3. Solve for x. ✓

$$\log_{10}\left(10^{x-1}\right) = \log_{10}(13)$$

$$x - 1 = \log(13)$$

$$+1 \qquad +1$$

$$\boxed{x = \log(13) + 1}$$

Solve using the equation $\boxed{A = A_0 e^{kt}}$ where A_0 is the initial amount present and *A* is the amount present at time *t*.

A = Future amount

A_0 = Initial Amount

K = rate

t = time

$e \approx 2.7$

(use e key in calc.)

I invest $2000 at an annual rate of 15% compounded continuously. How much money will I have after 2 years?

$A = ?$

$A_0 = 2000$

$K = 15\% \to .15$

$t = 2$

$$A = \underbrace{2000e^{((.15)(2))}}_{\text{Type into Calc.}}$$

$A \approx \$2,699.72$

Solve using the equation $A = A_0 e^{kt}$ where A_0 is the initial amount present and *A* is the amount present at time *t*.

Iodine 131 is a radioactive material with a decay rate of 8.7%. Assume that a scientist has a sample of 100 grams of iodine 131. How long (in days) will it take until 60 grams of iodine 131 is left?

$A = 60$

$A_0 = 100$

$K = -.087 \leftarrow (8.7\%)$

$t = ?$

$\frac{60}{100} = \frac{100}{100} \boxed{e^{-.087 \cdot t}}$

$\ln(0.6) = \ln e^{-.087t}$

$\frac{\ln(0.6)}{-.087} = \frac{-.087t}{-.087}$

$\boxed{t \approx 5.87 \text{ days}}$

<u>Solving Exponents</u>

1. Isolate the exponent
2. Rewrite to a Log
3. Solve for x.

Solve using the equation $A = A_0 e^{kt}$ where A_0 is the initial amount present and *A* is the amount present at time *t*.

If a chemist has 30 grams of a substance that has a half-life of 8 hours, how much will be there after 20 hours? (Hint: find the decay rate first)

A

B If a chemist has 30 grams of a substance that has a half-life of 8 hours, how much will be there after 20 hours? (Hint: find the decay rate first)

A

$A = 15$

$A_0 = 30$

$K = ?$

$t = 8$

$$\frac{15}{30} = \frac{30e^{8K}}{30}$$

$$\ln(.5) = \ln e^{8K}$$

$$\frac{\ln(.5)}{8} = \frac{8K}{8}$$

$$K \approx -0.0866$$

B

$A = ?$

$A_0 = 30$

$K = -0.0866$

$t = 20$

$$A = 30e^{-0.0866 \cdot 20}$$

↓ Type in Calc

$$\boxed{A = 5.31 \text{ grams}}$$