Lesson 9.1: Properties of Logarithms

common log: log a
Natural log: ln

Change of Base Formula:

$$\log_b a = \frac{\log a}{\log b} \text{ or } \frac{\ln a}{\ln b}$$

base 10 base e

Evaluate:

$\log_4 17 = \frac{\log 17}{\log 4} \approx 2.04$

$\log_8 3 = \frac{\log(3)}{\log(8)} \approx 0.53$

$8^x = 3$

$\log_5 125 - \log_5 5$

$5^x = 125$ | $5^x = 5$

$x = 3$ | $x = 1$

$3 - 1 = \boxed{2}$

$\Rightarrow \log_5 \frac{25}{x}$

$5^x = 25$

$x = 2$

$\log_5 125 + \log_5 5$

$x = 3$ $\quad$ $x = 1$

$3 + 1 = 4$

$\log_5 625$

$5^x = 625$

$x = 4$

Other Log Properties

$$\log_b x + \log_b y = \log_b xy$$

$$\log_b x - \log_b y = \log_b \frac{x}{y}$$

$$\log_b x^n = n\log_b x$$

Expand.

$\log_4 16x^2 = \log_4 16 + \log_4 x^2$

$\log_4 16 + 2\log_4 x$

$4^x = 16$

$2 + 2\log_4 x$

Expand.

$$\log_b x + \log_b y = \log_b xy$$

$$\log_b x - \log_b y = \log_b \frac{x}{y}$$

$$\log_b x^n = n\log_b x$$

$$\log_3 \frac{4x^3}{y^2}$$

$\log_3 4 + \log_3 x^3 - \log_3 y^2$

$\log_3 4 + 3\log_3 x - 2\log_3 y$

$3^x = 4$

Can't be evaluated.

$$\log \frac{100x}{y^2 z^3} = \log 100 + \log x - (\log y^2 + \log z^3)$$

$10^x = 100$

$\log 100 + \log x - \log y^2 - \log z^3$

$2 + \log x - 2\log y - 3\log z$

Condense.

$2\log_5 4 - \log_5 x$

$\log_5 4^2 - \log_5 x$

$\log_5 16 - \log_5 x$

$\log_5 \left(\frac{16}{x}\right)$

$\log_b x + \log_b y = \log_b xy$

$\log_b x - \log_b y = \log_b \frac{x}{y}$

$\log_b x^n = n\log_b x$

$5\log_4 x + \frac{1}{2}\log_4 y - \log_4 17$

$\log_4 x^5 + \log_4 y^{\frac{1}{2}} - \log_4 17$

$\log_4 \left(\frac{x^5 \cdot y^{\frac{1}{2}}}{17}\right)$

$\log_4 \left(\frac{x^5\sqrt{y}}{17}\right)$

Condense.

$$\log_9 7 + \log_9 x - 2\log_9 y - \frac{1}{2}\log_9 z$$

$$\log_9 7 + \log_9 x - \log_9 y^2 - \log_9 z^{\frac{1}{2}}$$

$$\log_9 \left(\frac{7x}{y^2\sqrt{z}} \right)$$

$b^{\log_b x} = x$

Solving Logs:

1. Condense the Logs (maximum of one log per side)
2. Rewrite to a Exponent or drop both logs
3. Solve for x.

Solve.

$\log_4 x + \log_4 2 = \log_4 18$

Condense

$\log_4 2x = \log_4 18$

$\frac{2x}{2} = \frac{18}{2}$

$x = 9$

Other Solving Method

$\log_4 2x = \log_4 18$

$2x = 18$

$x = 9$

Solve.

$\log x - \log 3 = \log 5$

$\log \frac{x}{3} = \log 5$

$3 \cdot \frac{x}{3} = 5 \cdot 3$

$x = 15$

Other

$7^{\log_7 4x} = 7^2$

$4x = 7^2$

$2\log_7 2 + \log_7 x = 2$

$\log_7 2^2 + \log_7 x = 2$

$\log_7 4 + \log_7 x = 2$

$\log_7 4x = 2$

$7^2 = 4x$

$\frac{49}{4} = \frac{4x}{4}$

$x = 12.25$

Solve.

$\log_2 x + 2\log_2 3 - \log_2 4 = 5$

$\log_2 x + \log_2 3^2 - \log_2 4 = 5$

$\log_2 x + \log_2 9 - \log_2 4 = 5$

$\log_2\left(\frac{9x}{4}\right) = 5$

$2^5 = \frac{9x}{4}$

$4 \cdot 32 = \frac{9x}{4} \cdot 4$

$\frac{128}{9} = \frac{9x}{9} \rightarrow \boxed{x = \frac{128}{9}}$

Other (New)

$\log_2\left(\frac{9x}{4}\right) = 5$

$\frac{9x}{4} = 2^5$