Lesson 3.2: Factoring Using Synthetic Division

$3x^3 - 8x^2 + 3x + 2$

Grouping Method.

$x^2(3x-8)+1(3x+2)$

Not the same!

Grouping Method fails, so we must use synthetic division to factor.

Synthetic Division Steps

1. Find an x that makes the expression equal 0.
2. Place that value in the box and perform synthetic division.
3. Write your solution (using the quotient and the divisor)
4. Finish factoring, if possible.

If you have trouble finding an x-value that works, here's how to find possible solutions:

$$\pm\frac{factors\ of\ last\ number}{factors\ of\ first\ number}$$

$$3x^3 - 8x^2 + 3x + 2$$

$$\pm\frac{1,2}{1,3} = \boxed{\pm 1, \pm\frac{1}{3}, \pm 2, \pm\frac{2}{3}}$$

$$3x^3 - 8x^2 + 3x + 2$$

1 $x = 1$: $3(1)^3 - 8(1)^2 + 3(1) + 2 = 0$ ✓

2

$$\begin{array}{c|cccc}
 & x^3 & x^2 & x & c \\
1 & 3 & -8 & 3 & 2 \\
 & \downarrow & 3 & -5 & -2 \\
+ & & & & \\
\hline
 & 3 & -5 & -2 & \boxed{0}
\end{array}$$

opposite

$$(x - 1)(3x^2 - 5x - 2)$$

factor more (bottoms up or diagram)

$$\boxed{(x-1)(3x+1)(x-2)}$$

$x = 1$ $-1/3$ $x = 2$ ← All solutions that could have worked in step 1

$6x^3 + 11x^2 + 6x + 1$

1. $\boxed{x = -1}$: $6(-1)^3 + 11(-1)^2 + 6(-1) + 1 = 0$ ✓

2.
$$\begin{array}{r|rrrr} -1 & 6 & 11 & 6 & 1 \\ + & \downarrow & -6 & -5 & -1 \\ \hline & 6 & 5 & 1 & \boxed{0} \end{array}$$

3. $(x+1)(6x^2+5x+1)$

4. $\boxed{(x+1)(3x+1)(2x+1)}$

$x = -1 \qquad -1/3 \qquad -1/2$ ← Solutions that could have worked in step 1

1) $x^3 + 6x^2 + 11x + 6$

$x = -3$

$x = -2$

1) $x = -1$ ✓

2)

$$
\begin{array}{r|rrrr}
-1 & 1 & 6 & 11 & 6 \\
 & & -1 & -5 & -6 \\
\hline
 & 1 & 5 & 6 & \boxed{0}
\end{array}
$$

3) $(x+1)(x^2+5x+6)$

4) $\boxed{\underline{(x+1)}\,\underline{(x+2)}\,\underline{(x+3)}}$

$x^4 - 2x^3 + 10x^2 - 18x + 9$

① $\boxed{x=1}$ $(1)^4 - 2(1)^3 + 10(1)^2 - 18(1) + 9 = 0$ ✓

②

$$\begin{array}{c|ccccc} & x^4 & x^3 & x^2 & x & c \\ 1 & 1 & -2 & 10 & -18 & 9 \\ + & \downarrow & 1 & -1 & 9 & -9 \\ \hline & 1 & -1 & 9 & -9 & \boxed{0} \end{array}$$

③ $(x-1)(x^3 - 1x^2 + 9x - 9)$

$x^2(\underline{x-1}) + 9(\underline{x-1})$

④ $\boxed{(x-1)(x^2+9)(x-1)}$

11. $2x^4 + 5x^3 + 6x^2 + 4x + 1$

① $\boxed{x = -1}$ $\quad 2(-1)^4 + 5(-1)^3 + 6(-1)^2 + 4(-1) + 1 = 0$ ✓

②

$$
\begin{array}{r|rrrrr}
-1 & 2 & 5 & 6 & 4 & 1 \\
 & \downarrow & -2 & -3 & -3 & -1 \\
\hline
 & 2 & 3 & 3 & 1 & \boxed{0}
\end{array}
$$

③ $(x+1)(2x^3 + 3x^2 + 3x + 1)$

$x^2(2x+3) + 1(3x+1)$

↳ Grouping Method Fails

So, to finish ④ we have to use synthetic division again

To factor

$2x^3 + 3x^2 + 3x + 1$ $\quad \pm\frac{1}{2,1} = \pm 1, \pm\frac{1}{2}$

1. $x \neq -1$:

$2(-1)^3 + 3(-1)^2 + 3(-1) + 1 = -1$

$\boxed{x = -\frac{1}{2}}$ $\quad 2(-\frac{1}{2})^3 + 3(-\frac{1}{2})^2 + 3(-\frac{1}{2}) + 1 = 0$ ✓

2.
$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 3 & 3 & 1 \\ & & -1 & -1 & -1 \\ \hline & 2 & 2 & 2 & \boxed{0} \end{array}$$

3. $(x+1)(x+\frac{1}{2})(2x^2+2x+2)$

GCF: 2

4. $(x+1)(x+\frac{1}{2})\,2(x^2+x+1)$

$\boxed{(x+1)(2x+1)(x^2+x+1)}$

Does not factor more.

How to Know When to Use What Method

1. Grouping Method: 4 terms
2. Diagram/Bottoms Up: 3 terms (highest exponent is twice as big as middle term's exponent)
3. Perfect Squares: 2 terms

4: Perfect cubes: 2 terms

5. Synthetic Division: Every other method fails