Lesson 14.2: Geometric Sequences and Series

In a geometric sequence, the ratio between successive terms is constant.

Multiplication/Division

Find the 15th term

$$3, 6, 12, 24, \ldots$$

$\cdot 2 \quad \cdot 2 \quad \cdot 2$

a_n

$a_1 = 3$

$a_2 = 3(2)^1$

$a_3 = 3(2)(2) = 3(2)^2$

$a_4 = 3(2)^3$

$\vdots$

$a_{15} = 3(2)^{14} = 49{,}152$

Explicit

$$\boxed{a_n = a_1(r)^{n-1}}$$

$a_1 = 1^{st}$ term)

$r =$ common ratio

Find the 10th term.

$$\frac{2}{3}, -2, 6, -18, \ldots$$

$\cdot -3 \quad \cdot -3 \quad \cdot -3$

Trick

$\boxed{r = \frac{a_2}{a_1}}$

Explicit

$\boxed{a_n = \frac{2}{3}(-3)^{n-1}}$

10th

$a_{10} = \frac{2}{3}(-3)^{(10-1)}$

$\boxed{-13,122}$

Find the missing term.

$$\underbrace{7, \underline{\;21\;}}_{\cdot r}, \underbrace{\underline{\;21\;}, 63}_{\cdot r}$$

$$\frac{7r^2}{7} = \frac{63}{7}$$

$$\sqrt{r^2} = \sqrt{9}$$

$$r = 3$$

Find the missing terms.

$3 \; \boxed{2nd} \; \boxed{\wedge} \; \boxed{(} \; 1/8 \; \boxed{)}$

$$400, \underline{200}, \underline{100}, 50$$

$\cdot r \quad \cdot r \quad \cdot r$

$$\frac{400r^3}{400} = \frac{50}{400}$$

$$\sqrt[3]{r^3} = \sqrt[3]{1/8}$$

$$r = 1/2$$

Finding Sums of a Geometric Series

Finite

$$S_n = \frac{a_1 - r \cdot a_n}{1 - r}$$

a_1 = 1st term

r = common ratio

a_n = last term

Infinite

$$S = \frac{a_1}{1 - r}$$

$|r| < 1$

converges

* If $|r| > 1$ then the series diverges

$$1 + 2 + 4 + 8 + \cdots$$

Diverges $(r = 2)$

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$$

Converges $(r = 1/2)$

Find the Sum

$$5 + 10 + 20 + \cdots + \underline{10{,}240}$$

$\times 2$ $\times 2$

$$S_n = \frac{a_1 - r a_n}{1 - r} = \frac{(5 - 2 \cdot 10240)}{(1-2)}$$

$a_1 = 5$

$r = 2$

$a_n = 10{,}240$

$$\boxed{20{,}475}$$

Find the Sum

$$\sum_{k=1}^{13} \overset{a_1}{1024}\left(\frac{\overset{r}{1}}{2}\right)^{k-1}$$

$$S_n = \frac{(a_1 - r \cdot a_n)}{(1-r)}$$

$a_1 = 1024$

$r = 1/2$

$a_n = 1024\left(\frac{1}{2}\right)^{(13-1)} = 1/4$

$$\frac{\left(1024 - \frac{1}{2} \cdot \frac{1}{4}\right)}{\left(1 - \frac{1}{2}\right)} = \boxed{2047.75}$$

14 $40 + 20 + \cdots$

$$r = \frac{a_2}{a_1} = \frac{20}{40} = 0.5$$

Determine if it converges or diverges and then find the sum.

$|r|<1$ $|r|>1$

$$\sum_{k=1}^{\infty} 2(3)^{k-1}$$

$r=3$

$3>1$

$\boxed{\text{Diverges}}$

a_1

$$\sum_{k=1}^{\infty} 3\left(\frac{1}{4}\right)^{k-1}$$

$r=1/4$

$1/4<1$

$\boxed{\text{Converges}}$

$S=\dfrac{a_1}{1-r}$

$S=\dfrac{(3)}{(1-1/4)}=\boxed{4}$

Determine if it converges or diverges and then find the sum.

$$2+\frac{4}{3}+\frac{8}{9}+\cdots$$

Trick: $r=\frac{a_2}{a_1}$

$r=\frac{(4/3)}{(2)}=\frac{2}{3}$

$\boxed{\text{converges}}$

$S=\frac{a_1}{1-r}=\frac{2}{(1-2/3)}=\boxed{6}$

Write in summation notation

$$4096 + 1024 + 256 + \cdots + \frac{1}{16}$$

$r = \frac{a_2}{a_1}$

$r = \frac{1024}{4096} =$

$$\boxed{\sum_{n=1}^{9} 4096(0.25)^{n-1}}$$

$\frac{1}{16} = 4096(0.25)^{n-1}$

$\downarrow$

0.0625

Guess & Check

$n = 9$