Lesson 14.1: Arithmetic Sequences and Series

In an arithmetic sequence, the difference between successive terms is constant (or the same number).

Addition / Subtraction

Find the 43rd term...

1 2

$4, 7, 10, 13, 16, \ldots$

+3 +3

1st: 4

2nd: $4+3(1)$

3rd: $4+3+3 = 4+3(2)$

4th: $4+3+3+3 = 4+3(3)$

5th: $4+3+3+3+3 = 4+3(4)$

common difference

43rd : $4+3(42)$

$= 130$

Explicit Formula

$a_n = a_1 + d(n-1)$

a_1 = First term

d = Common difference

Find the 27th term.

$$a_n = a_1 + d(n-1)$$

$$110, 99, 88, 77, \ldots$$

$-11 \quad -11 \quad -11$

$a_1 = 110$

$d = -11$

Explicit:

$$a_n = 110 - 11(n-1)$$

27th Term:

$$a_{27} = 110 - 11(27-1)$$

$$a_{27} = -176$$

Find the missing term

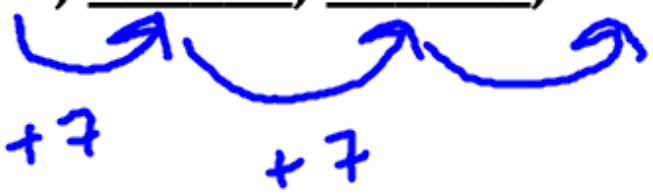
35, 31 ,27

d d

$d = \frac{(27-35)}{2} = \frac{-8}{2} = -4$

Find the missing term

19, 26, 33, 40

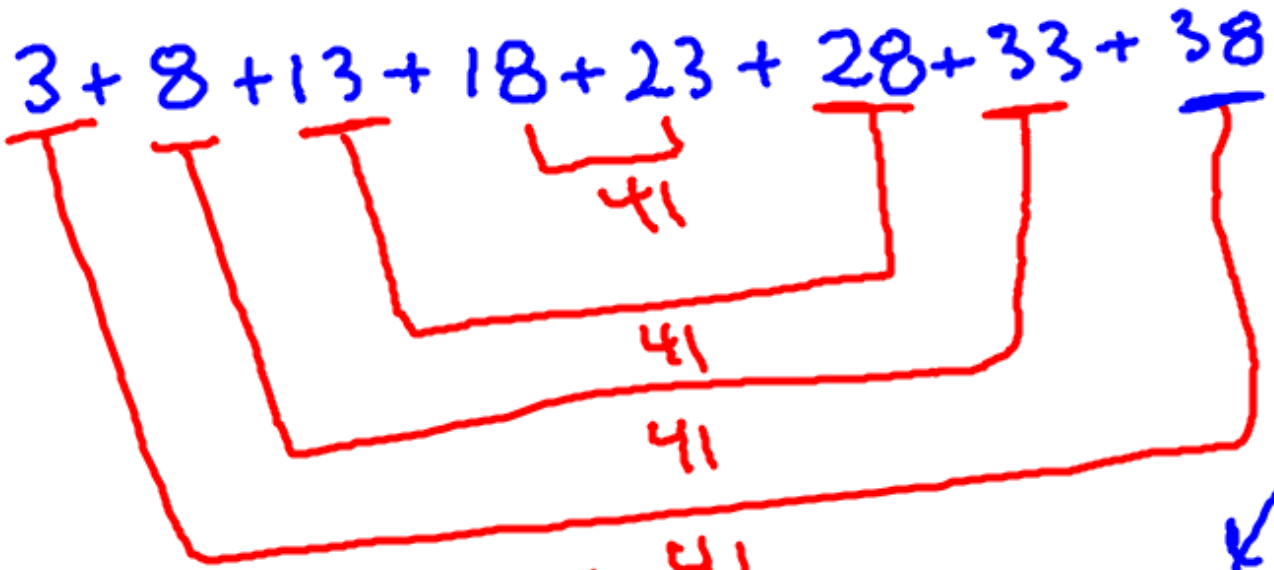


$d = \frac{40-19}{3} = \frac{21}{3} = 7$

Find the Sum.

+5 +5

$$3 + 8 + 13 + \cdots + 38$$



8 terms

Paired off =

4 pairs

$$\frac{8}{2}(3 + 38)$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

n: number of terms

a_1: 1st term

a_n: Last term

Find the sum of $-1 + 3 + 7 + \cdots + 151$

+4 +4

$a_n = -1 + 4(n-1)$

$151 = -1 + 4(n-1)$

$\frac{152}{4} = \frac{4(n-1)}{4}$

$38 = n-1$

$n = 39$ terms

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$S_n = \frac{39}{2}(-1 + 151)$

$S_n = 2925$

#17

(17) $17+25+33+\cdots+177$

$a_n = 17+8(n-1)$

$177 = 17+8(n-1)$

$-17 \quad -17$

$\frac{160}{8} = \frac{8(n-1)}{8}$

$20 = n-1$

$+1 \quad +1$

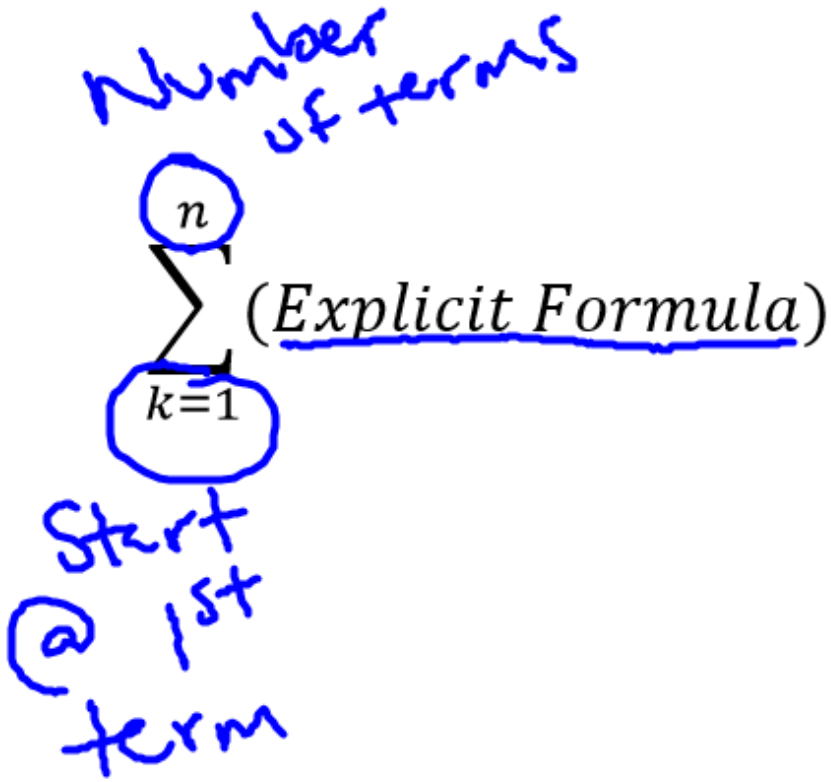
$n = 21$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_n = \frac{21}{2}(17+177)$

$S_n = 2037$

Summation Notation



$$\sum_{k=1}^{5} 2k+3$$

1st to 5th term

$a_1 = 2(1)+3 = 5$

$a_2 = 2(2)+3 = 7$

$a_3 = 2(3)+3 = 9$

$a_4 = 2(4)+3 = 11$

$a_5 = 2(5)+3 = 13$

$+$

45

$$\sum_{k=1}^{35} -3k + 1$$

$S_n = \frac{n}{2}(a_1 + a_n)$

$n = 35$

$a_1 = -3(1) + 1 = -2$

$a_n = -3(35) + 1 = -104$

$S_n = \frac{35}{2}(-2 + -104)$

$S_n = -1855$

Write the following in summation notation $8 + 11 + 14 + \cdots + 68$

$$\sum_{k=1}^{n} (Explicit\ Formula)$$

+3 +3

$$\sum_{n=1}^{21} 8+3(n-1)$$

Number terms

(Last term)

$68 = 8+3(n-1)$

$-8 \quad -8$

$\frac{60}{3} = \frac{3(n-1)}{3}$

$20 = n-1$

$n = 21$

Sequence vs. Series

, +

Infinite vs. Finite

ends with Final number

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