Lesson 13.4: Confidence Intervals

A more **general** formula for confidence interval: Statistic $\pm$ (**critical value**)(**standard deviation of statistic**).

The **critical value** is a **multiplier** that makes the interval **wide enough** to have the stated **capture rate (confidence level)**. The critical value **depends** both on the **confidence level C** and **sampling distribution** of the statistic.

Table of Critical Values for Given Confidence Intervals

Confidence Level	50%	60%	70%	80%	90%	95%	99%
Critical Value z^*	0.674	0.841	1.036	1.282	1.645	1.96	2.576

Confidence Interval for Proportions: $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Confidence Interval for Means: $\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$

$\hat{p}$ = sample proportion
z^* : critical value
n = sample size
$\bar{x}$: sample mean
σ = standard deviation

The small round holes you often see in sea shells were drilled by other sea creatures, who ate the former dwellers of the shells. Whelks often drill into mussels but this behavior appears to be more or less common in different locations. Researchers collected whelk eggs from the coast of Oregon, raised the whelks in the laboratory, and then put each whelk in a container with some delicious mussels. Only 9 of 98 whelks drilled into a mussel. The researchers want to estimate the proportion of Oregon whelks that will spontaneously drill into mussels (95% confidence level).

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$\hat{p} = \frac{9}{98}$

$z^* = 1.96$

$n = 98$

$$\frac{9}{98} \pm 1.96 \sqrt{\frac{9/98(1-9/98)}{98}}$$

$$(0.035 \ , \ 0.149)$$

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Tonya wants to estimate what proportion of her school's seniors plan to attend the prom. She interviews an SRS of 50 of the 750 seniors in her school and finds that 36 plan to go to the prom. Construct a 90% confidence interval for the proportion that will attend prom.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$\hat{p} = \frac{36}{50} = 0.72$

$z^* = 1.645$

$n = 50$

$$0.72 \pm 1.645\sqrt{\frac{0.72(1-0.72)}{50}}$$

$(0.616, 0.824)$

Tonya suspected that it would be around 80%. Does the confidence interval show evidence of Tonya's belief that it is around 80%? Explain.

Yes. 80% is within the interval.

Environmentalists, government officials, and vehicle manufacturers are all interested in studying the auto exhaust emissions produced by motor vehicles. The major pollutants in auto exhaust from gasoline engines are hydrocarbons, carbon monoxide, and nitrogen oxides (NOX). Researchers collected data on the NOX levels (in grams/mile) for a ~~random~~ sample of 40 light-duty engines of the same type. The sample mean NOX reading was 1.2675 ($\sigma = 0.3332$). Construct a 99% confidence interval for the mean NOX level.

$$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$$

$$1.2675 \pm 2.576 \cdot \frac{0.3332}{\sqrt{40}}$$

$\bar{x} = 1.2675$

$z^* = 2.576$

$\sigma = 0.3332$

$n = 40$

$(1.132, 1.403)$

In California, a low emissions vehicle has NOX level below 0.7. Is there evidence that these light-duty engines could be from low emissions vehicles? Explain.

No. 0.7 is not within the interval.

A bunion on the big toe is fairly uncommon in youth and often requires surgery. Doctors used X-rays to measure the angle (in degrees) of deformity on the big toe in a random sample of 37 patients under the age of 21 who came to a medical center for surgery to correct a bunion. The angle is a measure of the seriousness of the deformity. For these 37 patients the mean angle of deformity was 24.76 ($\sigma = 6.34$). Construct a 90% confidence interval for the mean angle of deformity in the population of such patients.

$$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$$

$\bar{x} = 24.76$

$z^* = 1.645$

$\sigma = 6.34$

$n = 37$

$$24.76 \pm 1.645 \cdot \frac{6.34}{\sqrt{37}}$$

$$(23.045,\ 26.475)$$