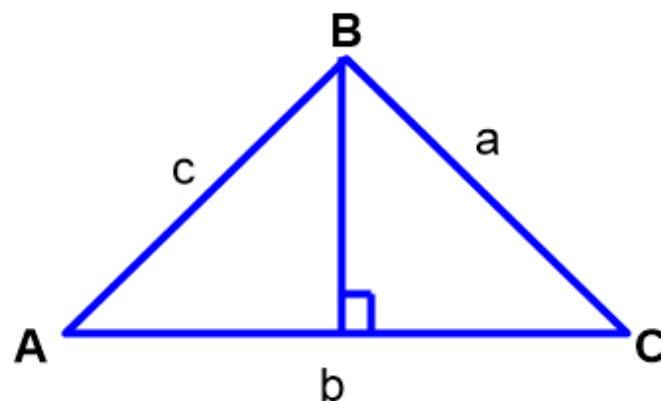


Lesson 11.3: Law of Sines and Area

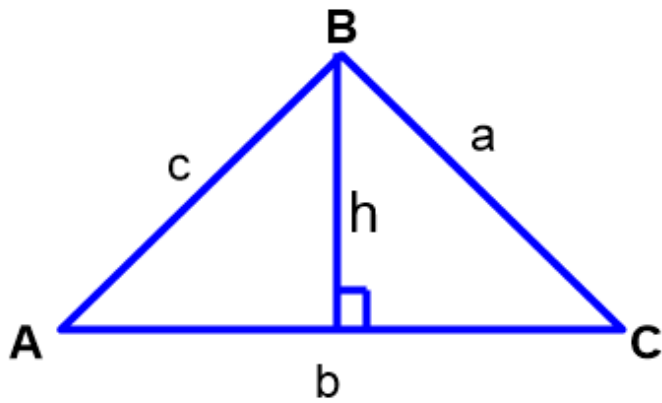
Sin (angle)
Opposite
Side



Law of Sines

$$\star \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

- Lowercase letters are sides.
- Uppercase Letters are angles.



Proof:

$$\sin A = \frac{h}{c}$$

$$\text{So } h = c \cdot \sin A$$

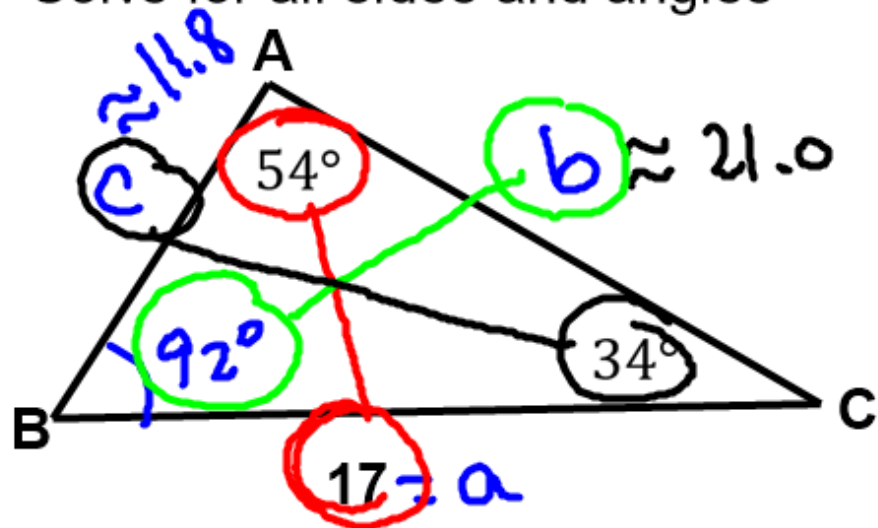
$$\sin C = \frac{h}{a}$$

$$\text{So } h = a \cdot \sin C$$

$$\text{Therefore, } c \cdot \sin A = a \cdot \sin C$$

$$\text{So, we can conclude that } \frac{\sin A}{a} = \frac{\sin C}{c}.$$

Solve for all sides and angles



$$B = 180 - 54 - 34 = \boxed{92^\circ}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Side b:

$$\frac{\sin(54^\circ)}{17} = \frac{\sin(92^\circ)}{b}$$

$$17 \sin(92) = b \sin(54)$$

$$b = \frac{17 \sin(92)}{\sin(54)} \approx \boxed{21.0}$$

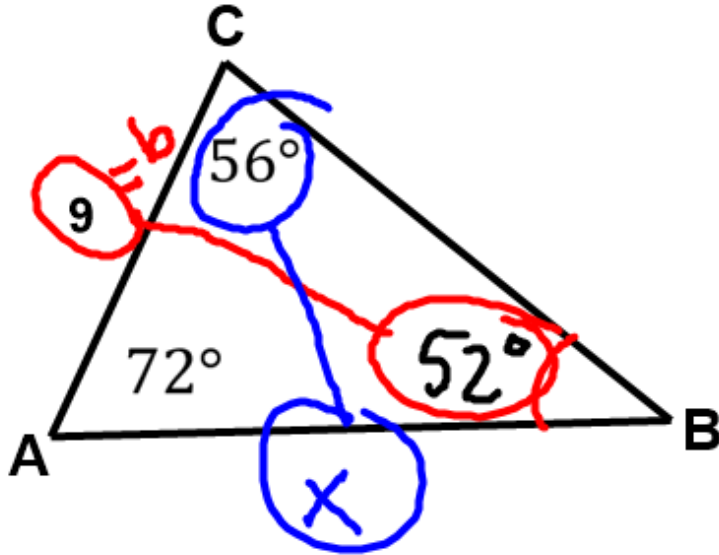
Side c:

$$\frac{\sin(54)}{17} = \frac{\sin(34)}{c}$$

$$17 \sin(34) = c \sin(54)$$

$$c = \frac{17 \sin(34)}{\sin(54)}$$
$$\boxed{c \approx 11.8}$$

Solve for all sides and angles ~~x~~



$$\frac{\sin(52^\circ)}{9} = \frac{\sin(56)}{x}$$

$$9 \sin(56) = x \sin(52)$$

$$x = \frac{9 \sin(56)}{\sin(52)}$$

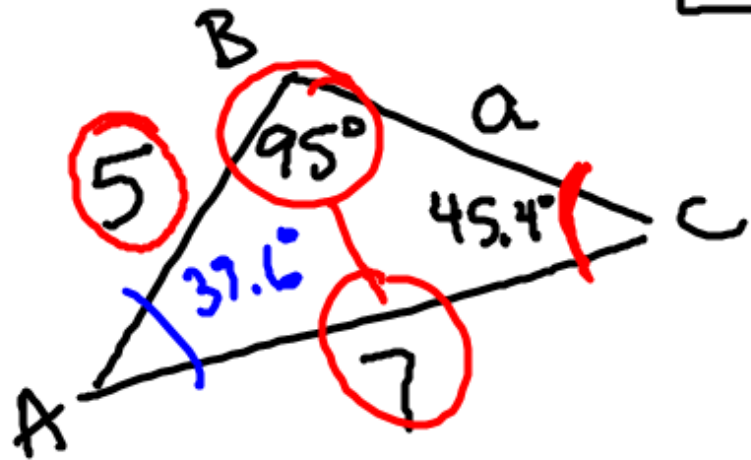
$$x \approx 9.5$$

Find B:

$$B = 180 - 56 - 72 = 52^\circ$$

Solve for all sides and angles

$$m\angle B = 95^\circ, b = 7, c = 5$$



$$5 \cdot \frac{\sin(95)}{7} = \frac{\sin C}{5} \cdot 5$$

$$\sin C = \frac{5 \sin(95)}{7}$$

$$C = \sin^{-1}\left(\frac{5 \sin(95)}{7}\right)$$

$$C \approx 45.4^\circ$$

$$A = 180 - 95 - 45.4$$

$$A = 39.6^\circ$$

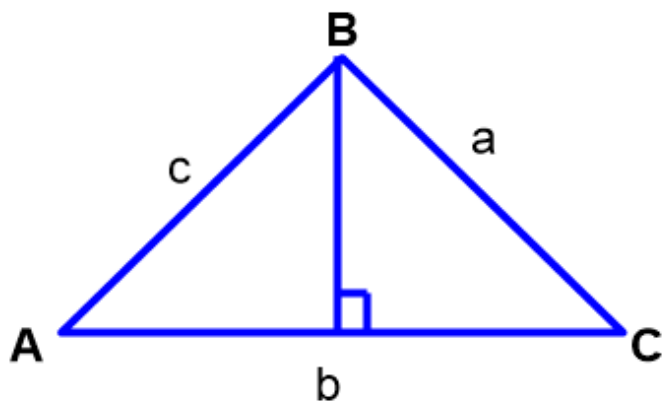
$$\frac{\sin(39.6)}{a} = \frac{\sin(95)}{7}$$

$$a \sin(95) = 7 \sin(39.6)$$

$$a = \frac{7 \sin(39.6)}{\sin(95)}$$

$$a \approx 4.5$$

Area of a Triangle (SAS)



$$\sin C = \frac{h}{a}$$

$$\text{So } h = a \cdot \sin C$$

$$\frac{1}{2}bh$$

↓

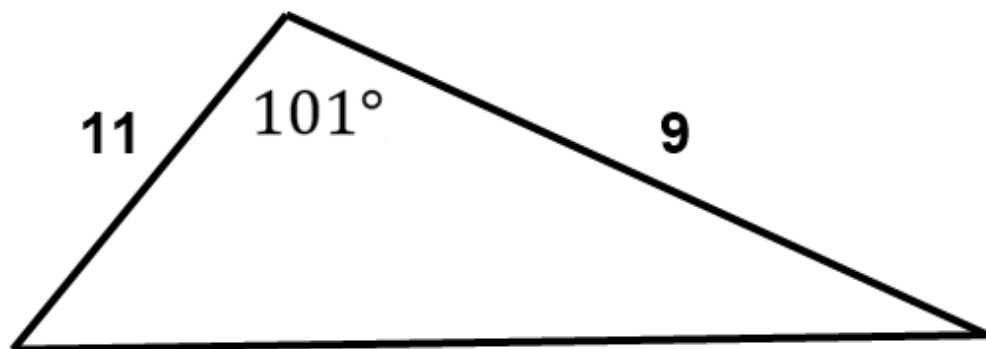
* $\frac{1}{2}ab \cdot \sin C$

↑ two sides

↑ angle between sides

*

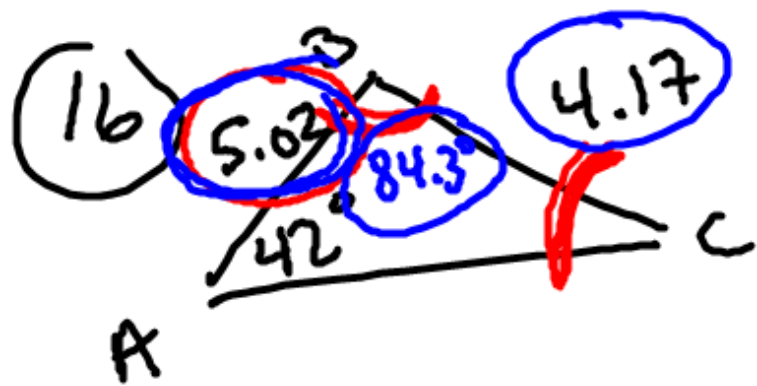
Find the Area



$$\frac{1}{2} ab \sin C$$

$$\frac{1}{2} (11)(9) \sin (101)$$

$$= 48.6$$



$$B = 180 - 42 - 53.7$$

$$B = 84.3$$

$$\frac{\sin C}{5.02} = \frac{\sin(42)}{4.17}$$

$$\sin C = \frac{5.02 \sin(42)}{4.17}$$

$$C = \sin^{-1}\left(\frac{5.02 \sin(42)}{4.17}\right)$$

$$C \approx 53.7^\circ$$

$$\frac{1}{2}(5.02)(4.17)\sin(84.3)$$

$$\boxed{10.4}$$