Lesson 11.1: Angles and the Unit Circle

Angle of Rotation

Initial Side

Direction of rotation

Vertex   Terminal Side
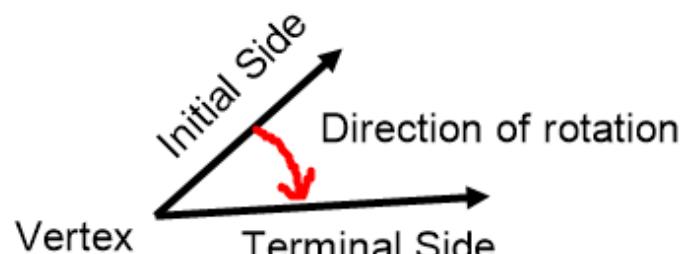
*This angle is not in standard position

* Standard Position: Vertex is at the origin and the initial side is on the positive x-axis.
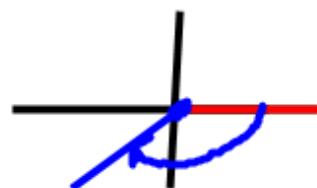
Negative Angle

* Counter-clockwise: Positive

* Clockwise: Negative

* Angles are **coterminal** when they share the same initial and terminal side.
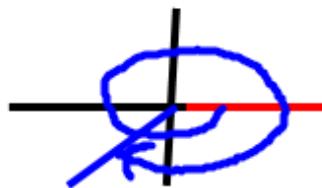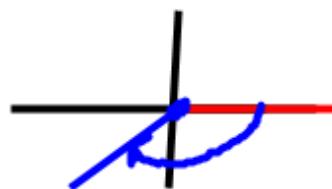
Find a negative and positive coterminal angle for the given angle.

* 106°

Positive

$106° + 360° = \boxed{466°}$

Negative

$106 - 360° = \boxed{-254°}$

* 415°

Positive:

$415° + 360° = \boxed{775°}$

*

Negative:

(+) $415° - 360° = 55°$

$55 - 360 = \boxed{-305°}$

y

(,)

(-, +)
II

90°

(,)
120°

(,)
135°

(,)
150°

(,)
180°

(+, +)
I

(,)
60°

(,)
45°

(,)
30°

(,)
0°
360°

(,)

x

(,)
210°

(,)
225°

(,)
240°

(,)
270°

330°
(,)

315°
(,)

300°
(,)

III
(-, -)

IV
(+, -)
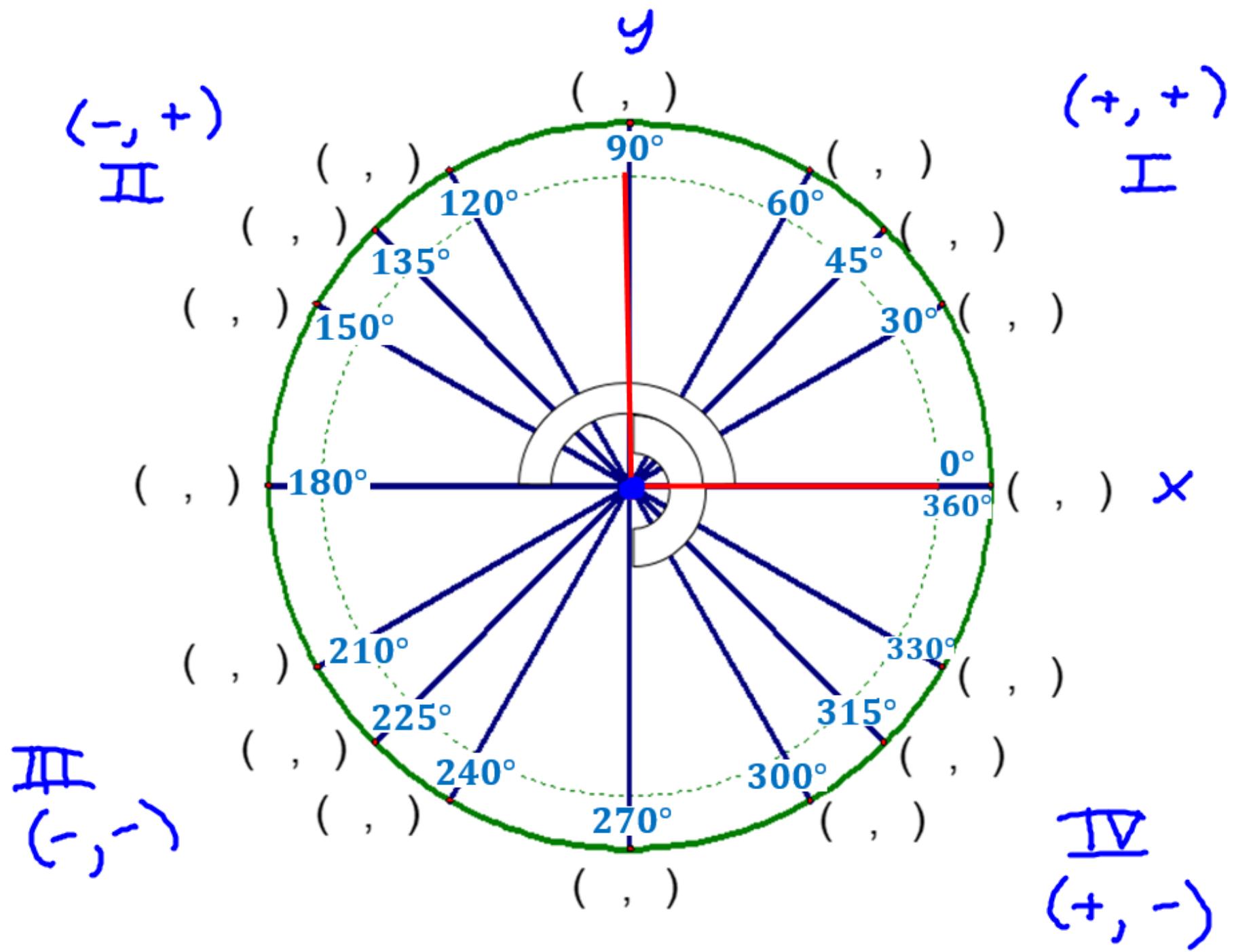
Determine the quadrant or axis where the terminal side of angle lies.

* $788°$

-360
-360
___
$68°$ $\boxed{I}$

* $450°$

-360
___
$90°$

$\boxed{y-axis}$

*

Quick Review

Trig Ratios

$$\sin \theta = \frac{opp}{hyp}$$

$$\cos \theta = \frac{adj}{hyp}$$

$$\tan \theta = \frac{opp}{adj}$$

Opposite

Hypotenuse = 1

θ

Adjacent

$(0, 1)$

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ 90° $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ 120° 60° $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ 135° 45° $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

150° 30°

$(-1, 0)$ 180° 0° $(1, 0)$

360°

210° 330°

$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

225° 315°

$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

240° 300°

$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ 270° $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$(0, -1)$
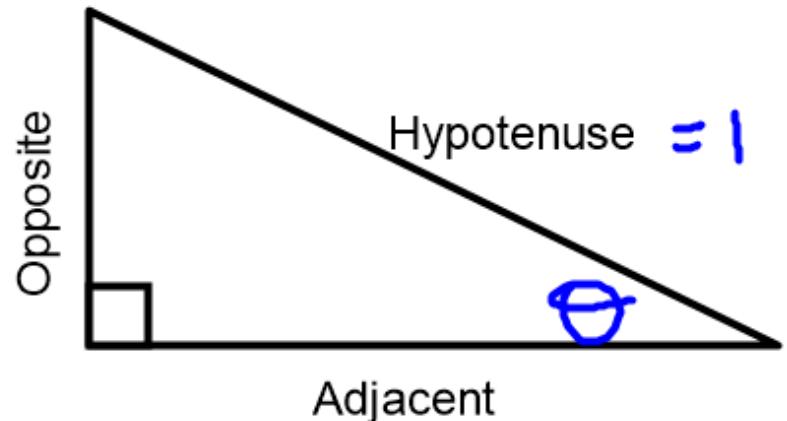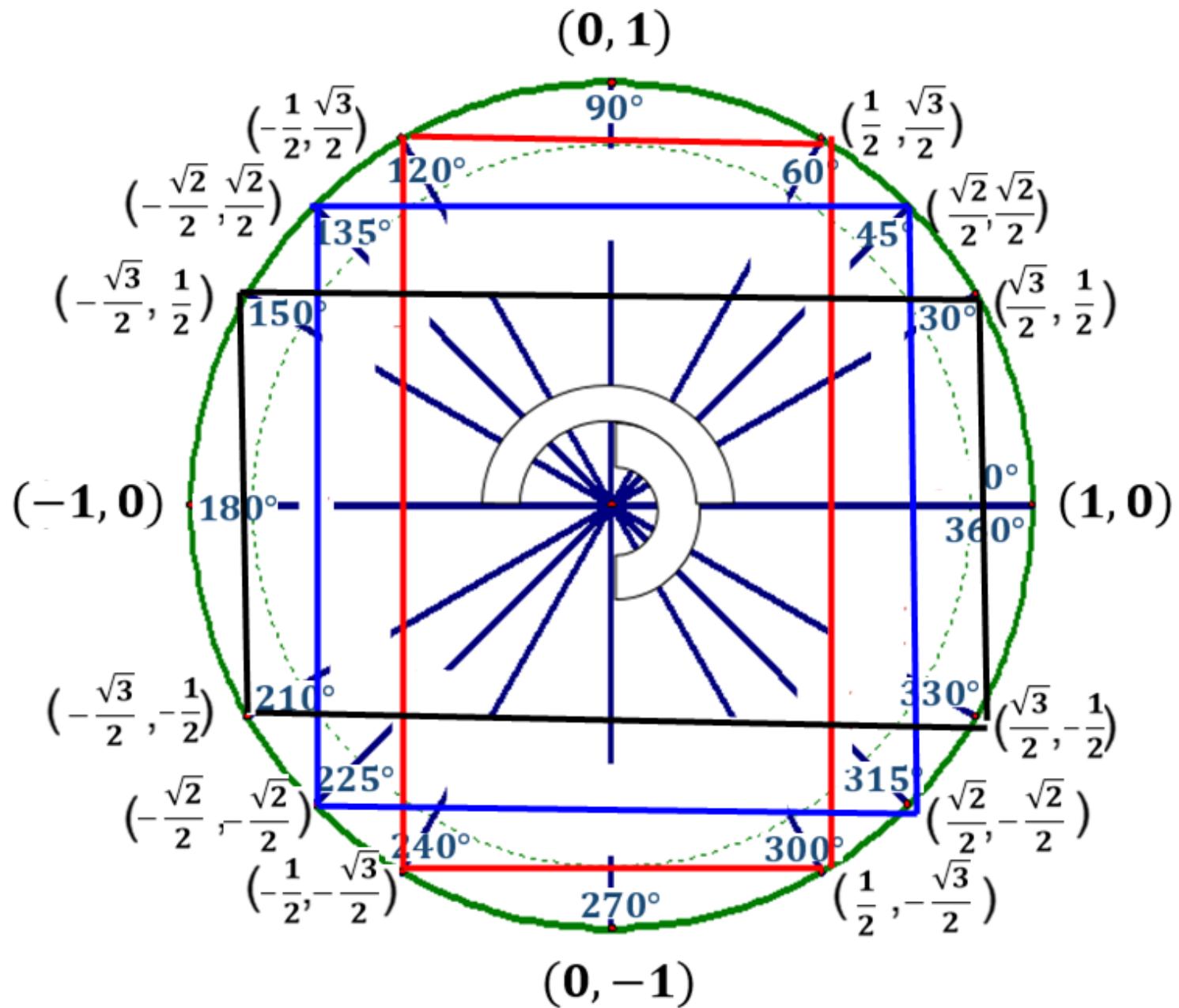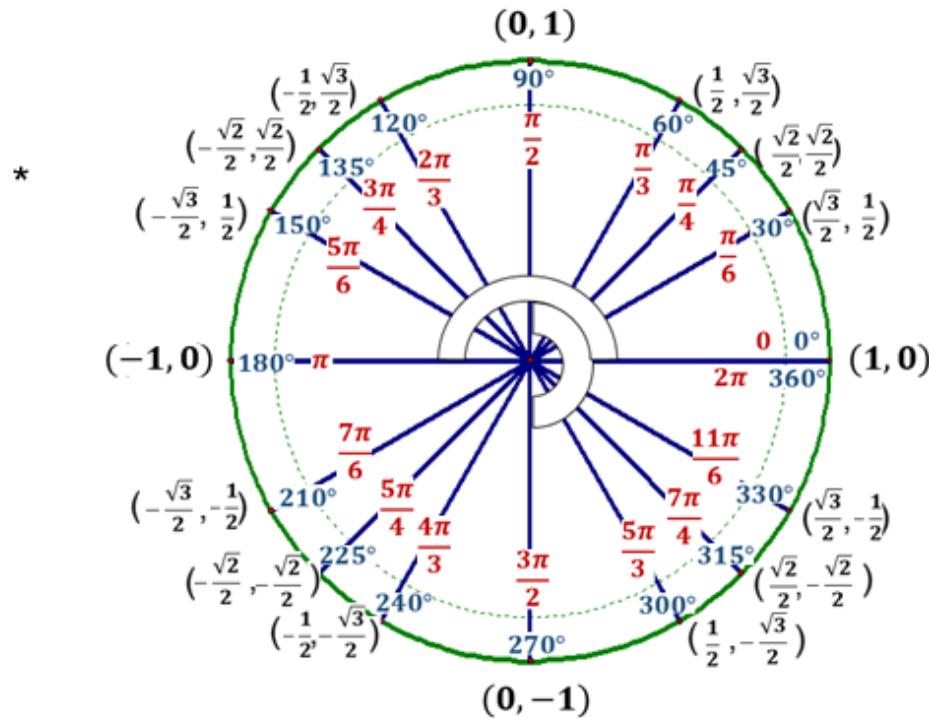
| $\cos\theta =$ x	$\sin\theta =$ y	$\tan\theta =$ $\dfrac{y}{x}$
$\sec\theta =$ $\dfrac{1}{x}$	$\csc\theta =$ $\dfrac{1}{y}$	$\cot\theta =$ $\dfrac{x}{y}$

Sketch the angle in standard position. Use the Unit Circle to find the exact values for the given trigonometric functions.

$$\sin 210° = \boxed{-\frac{1}{2}}$$
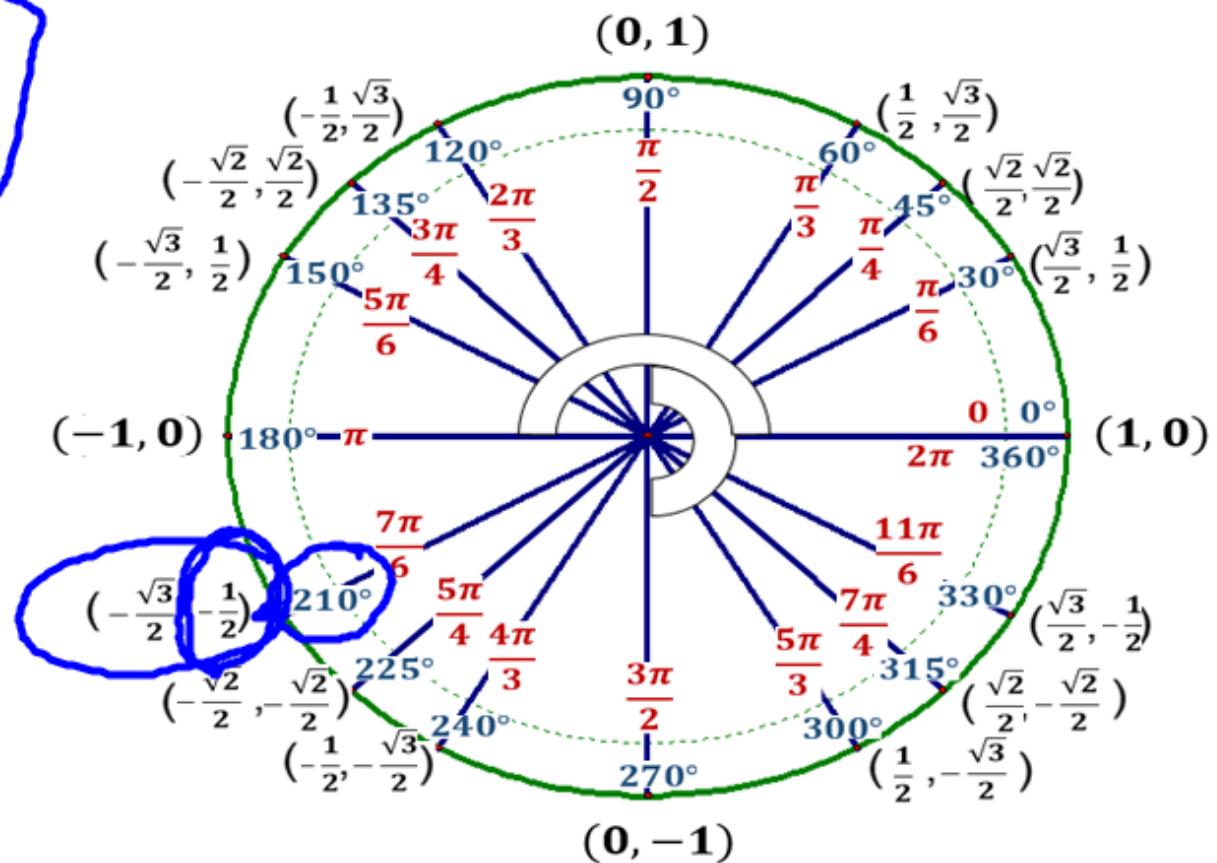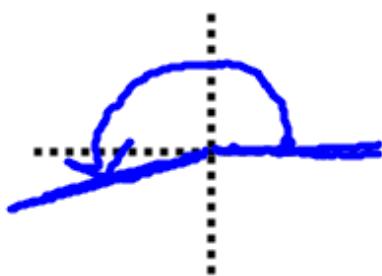
Sketch the angle in standard position. Use the Unit Circle to find the exact values for the given trigonometric functions.

$$\cos(-45°) = \boxed{\dfrac{\sqrt{2}}{2}}$$

*
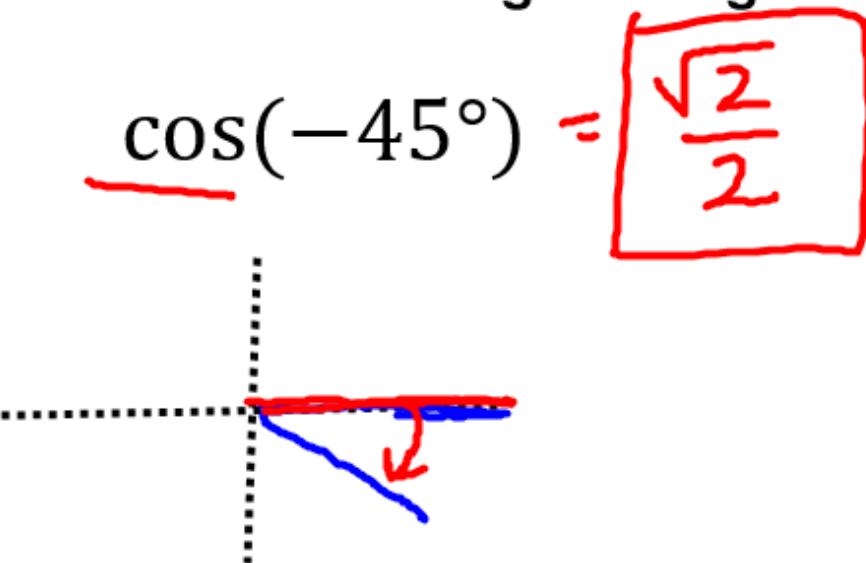
$-45 + 360 = 315°$
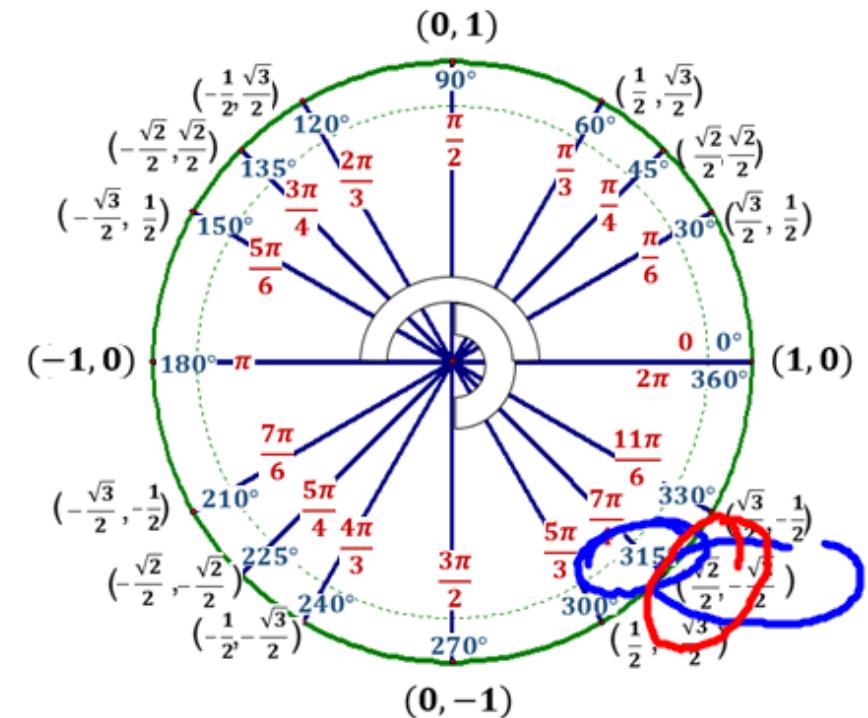
**Sketch the angle in standard position. Use the Unit Circle to find the exact values for the given trigonometric functions.**

$$\sin 495° = \frac{\sqrt{2}}{2}$$
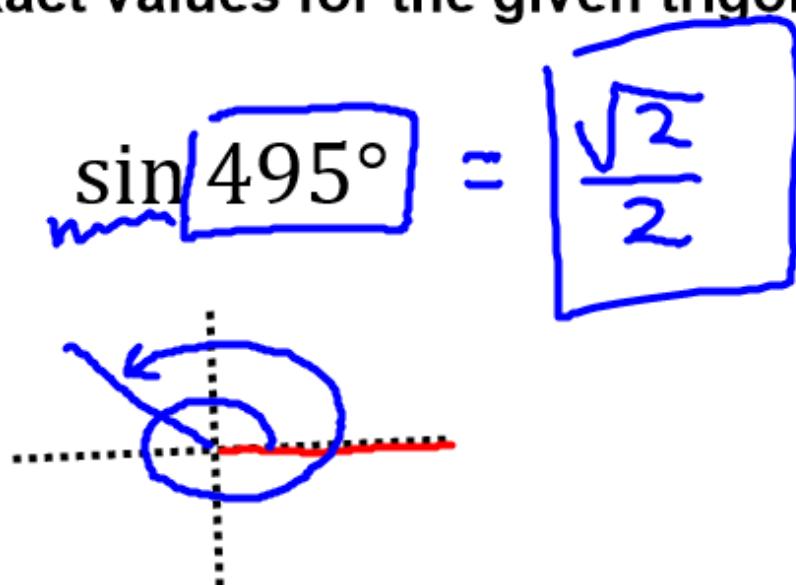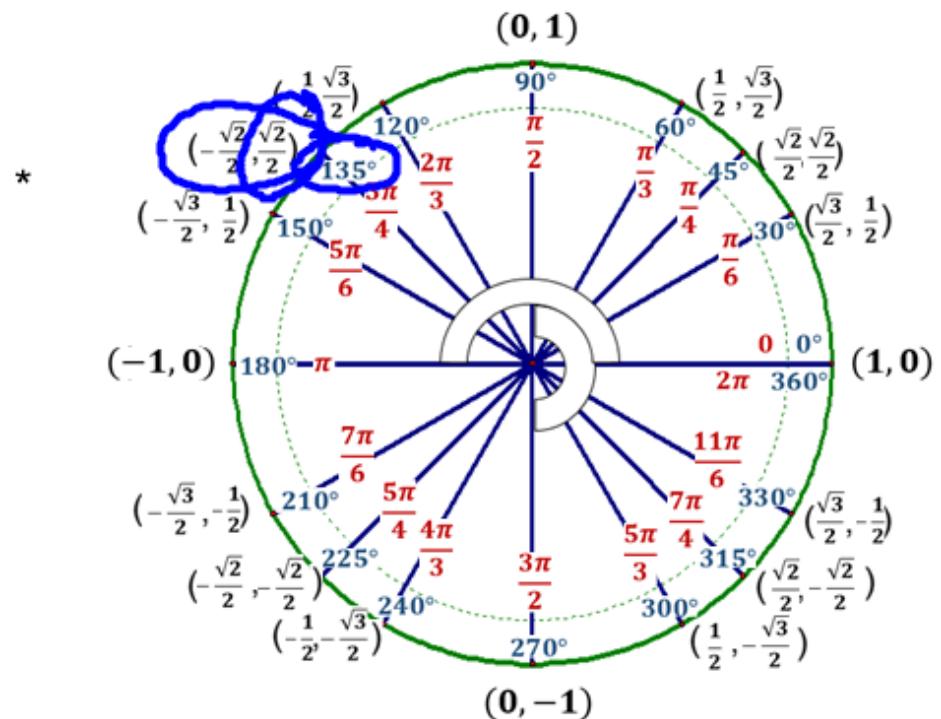
$$495 - 360 = 135°$$

Sketch the angle in standard position. Use the Unit Circle to find the exact values for the given trigonometric functions.

$$\tan 45° = \frac{y}{x} = \frac{\sqrt{2}/2}{\sqrt{2}/2} = \boxed{1}$$
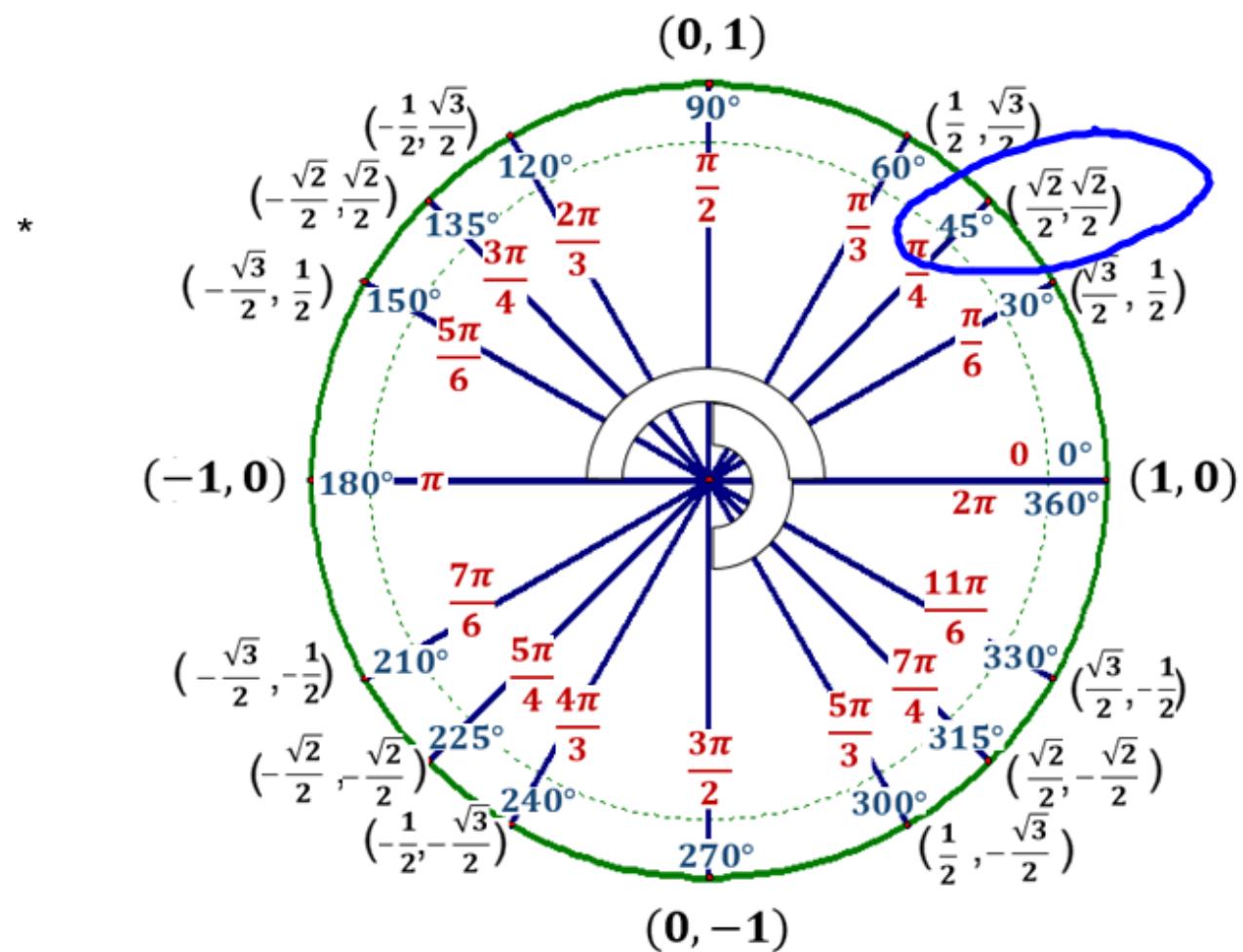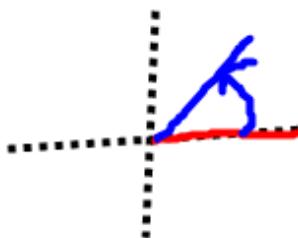
Sketch the angle in standard position. Use the Unit Circle to find the exact values for the given trigonometric functions.

$$\tan \boxed{450°} = \frac{y}{x} = \frac{1}{0} = \boxed{\text{undefined}}$$

$450 - 360 = 90°$
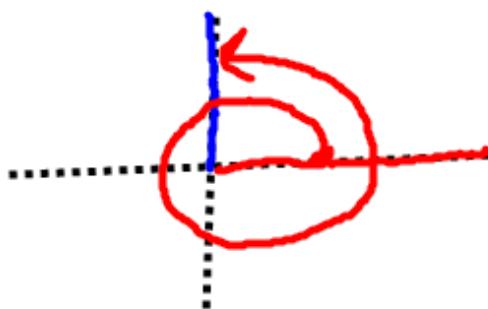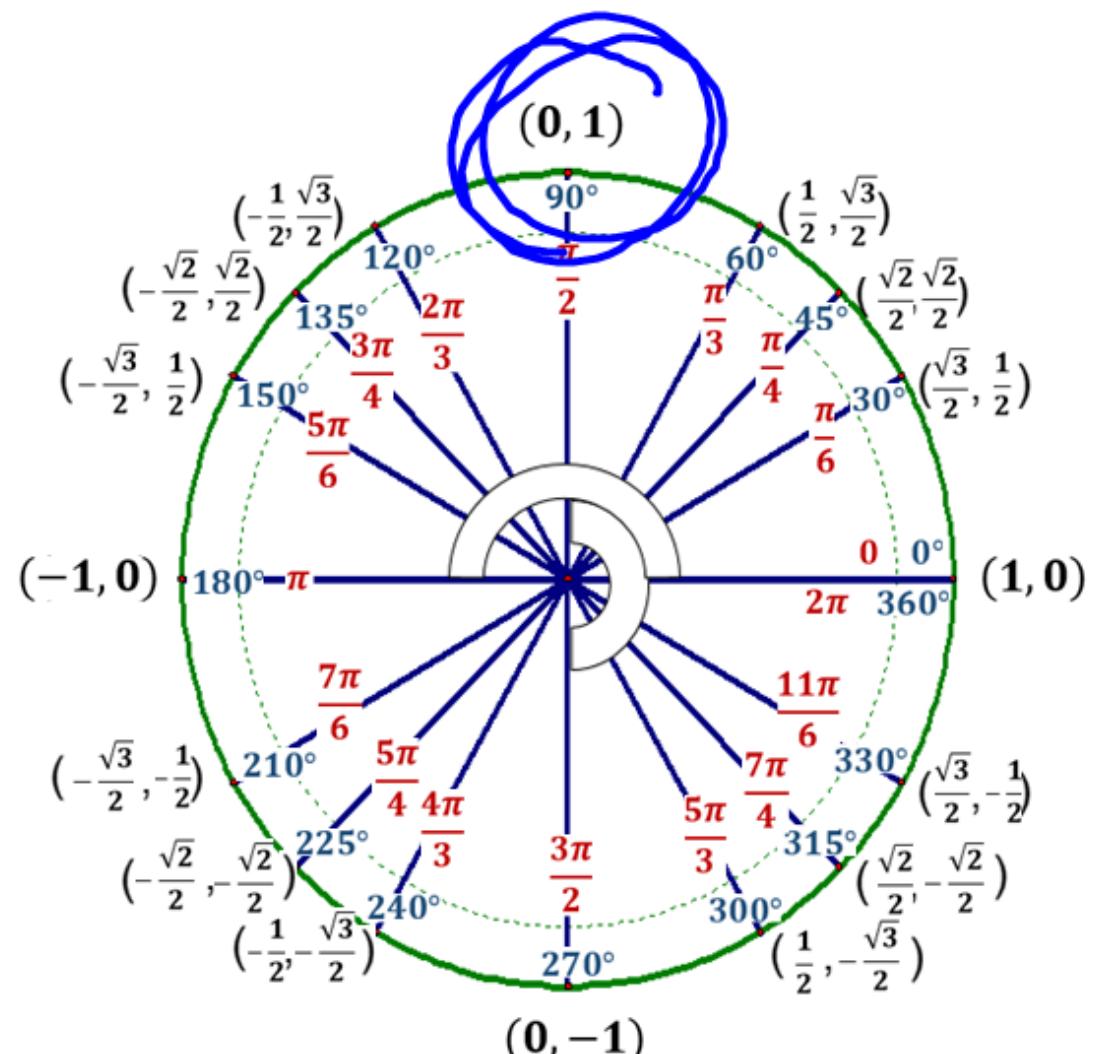
(0, 1)

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ 90° $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

120°   60°

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ 135° $\frac{\pi}{2}$ 45° $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\frac{2\pi}{3}$ $\frac{\pi}{3}$ $\frac{\pi}{4}$

$\frac{3\pi}{4}$

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ 150°   30° $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\frac{5\pi}{6}$ $\frac{\pi}{6}$

$(-1, 0)$ 180°—π 0   0° $(1, 0)$

2π 360°

$\frac{7\pi}{6}$ $\frac{11\pi}{6}$

$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ 210°   330° $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\frac{5\pi}{4}$ $\frac{4\pi}{3}$ $\frac{5\pi}{3}$ $\frac{7\pi}{4}$

$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ 225° $\frac{3\pi}{2}$ 315° $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

240°   300°

$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ 270° $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

(0, −1)

Sketch the angle in standard position. Use the Unit Circle to find the exact values for the given trigonometric functions.

$$\tan(-120)° = \frac{y}{x} = \frac{-\sqrt{3}/2}{-1/2} \cdot \frac{2}{2} = \frac{\sqrt{3}}{1} = \boxed{\sqrt{3}}$$

$-120 + 360 = 240°$