

# **Lesson 1.1: Functions, Domain and Average Rate of Change**

Honors Math 3

Ex 1: Find the following values for each function:

$$f(x) = \frac{2x}{x^2 + 4}$$

- a)  $f(-2)$       b)  $f(\underline{-x})$       c)  $-f(x)$       d)  $f(x + 3)$

(a)  $f(-2) = \frac{2(-2)}{(-2)^2 + 4} = \frac{-4}{4+4} = \frac{-4}{8} = -\frac{1}{2}$

(b)  $f(-x) = \frac{2(-x)}{(-x)^2 + 4} = \frac{-2x}{x^2 + 4}$

Ex 1: Find the following values for each function:

$$f(x) = \frac{2x}{x^2 + 4}$$

a)  $f(-2)$

b)  $f(-x)$

c)  $-f(x)$

d)  $f(x + 3)$

(c)  $-1 \left( \frac{2x}{x^2 + 4} \right) = -\frac{2x}{x^2 + 4}$

$$-\frac{1}{2} = \frac{-1}{2} = -\frac{1}{2}$$

Ex 1: Find the following values for each function:

$$f(x) = \frac{2x}{x^2 + 4}$$

a)  $f(-2)$

b)  $f(-x)$

c)  $-f(x)$

d)  $f(x+3)$

(d)

$$\frac{\cancel{2}(x+3)}{\cancel{(x+3)^2} + 4} = \frac{2x+6}{x^2+6x+9+4}$$

$$(x+3)(x+3) = \frac{2x+6}{x^2+6x+13}$$

Ex 2: Find the following values for each function:

$$h(x) = |x + 1|$$

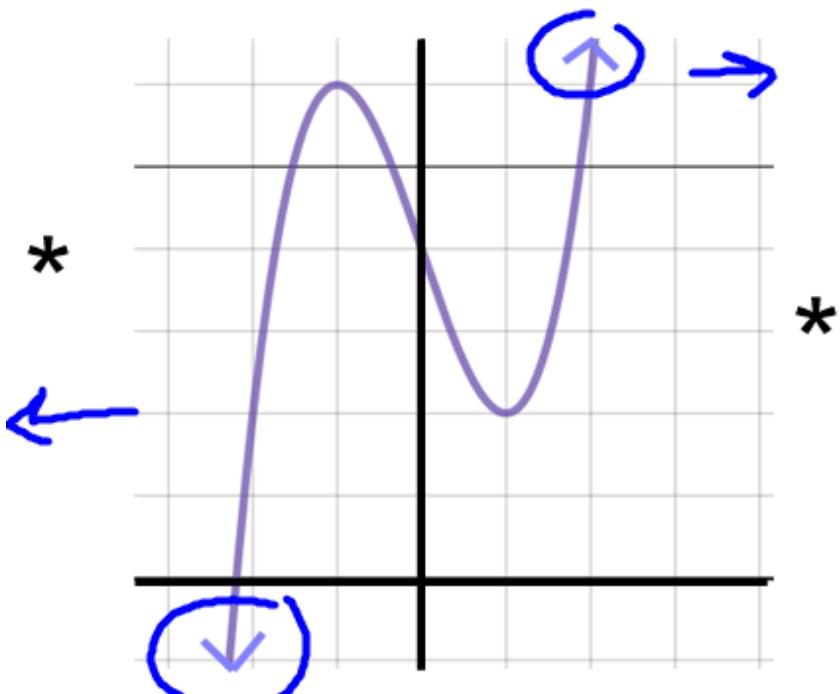
a)  $h(-5)$       b)  $h(-x + 3)$

a)  $h(-5) = |-5 + 1| = |-4| = \boxed{4}$

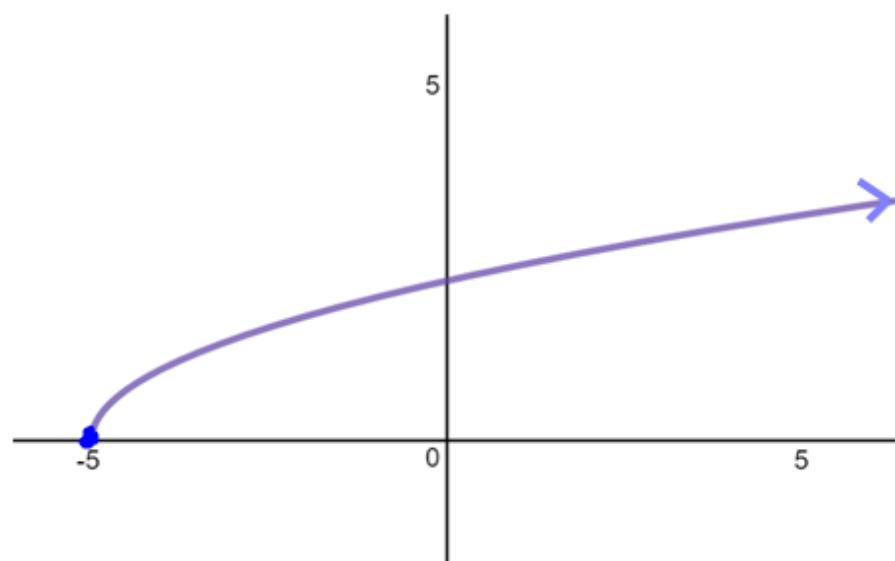
b)  $h(-x + 3) = |-x + 3 + 1| = |-x + 4|$

## What is the domain of a function?

- \* The set of all real number inputs which produce a valid (or real number) output.



$\mathbb{R}$   
 $(-\infty, \infty)$



$$D : [-5, \infty)$$
$$x \geq -5$$

Ex 3: Find of the domain of each function.

$$f(x) = 2x + 1$$

$$D : \mathbb{R}$$

$$h(x) = \sqrt{x + 1}$$

You can't  
square root  
a negative  
and get a  
real number.

$$x \geq -1$$

$$x + 1 \geq 0$$

$$D: x \geq -1$$

Ex 4: Find of the domain of each function.

$$g(x) = \sqrt{3x - 12}$$

$$3x - 12 \geq 0$$

$$\frac{3x}{3} \geq \frac{12}{3}$$

$$D: x \geq 4$$

$$j(x) = \frac{2}{x + 2}$$

Can't divide  
by 0.

$$D: x + 2 \neq 0$$

$$x \neq -2$$

Ex 5: Find of the domain of each function.

$$f(x) = \frac{2x}{x^2 - 4}$$

$$x^2 - 4 \neq 0$$

$$\sqrt{x^2} \neq \sqrt{4}$$

$$x \neq 2, -2$$

$$x \neq \pm 2$$

$$g(x) = \frac{x}{x^2 + 1}$$

$$x^2 + 1 \neq 0$$

$$\sqrt{x^2} \neq \sqrt{-1}$$

imaginary!

$$\frac{-1}{(-1)^2 + 1} = \frac{-1}{2}$$

$$D: \mathbb{R}$$

Ex 6: Find of the domain of each function.

$$h(x) = \frac{x}{\sqrt{x - 4}}$$

\*  
 $\sqrt{\phantom{x}}$        $\div$

$$\frac{x-4 > 0}{D: x > 4}$$

$$j(x) = x^3 + 5x + 4$$

$$X \cdot X \cdot X$$

$$D: \mathbb{R}$$

## Summary: How to Find the Domain of a Function

### Red Flags

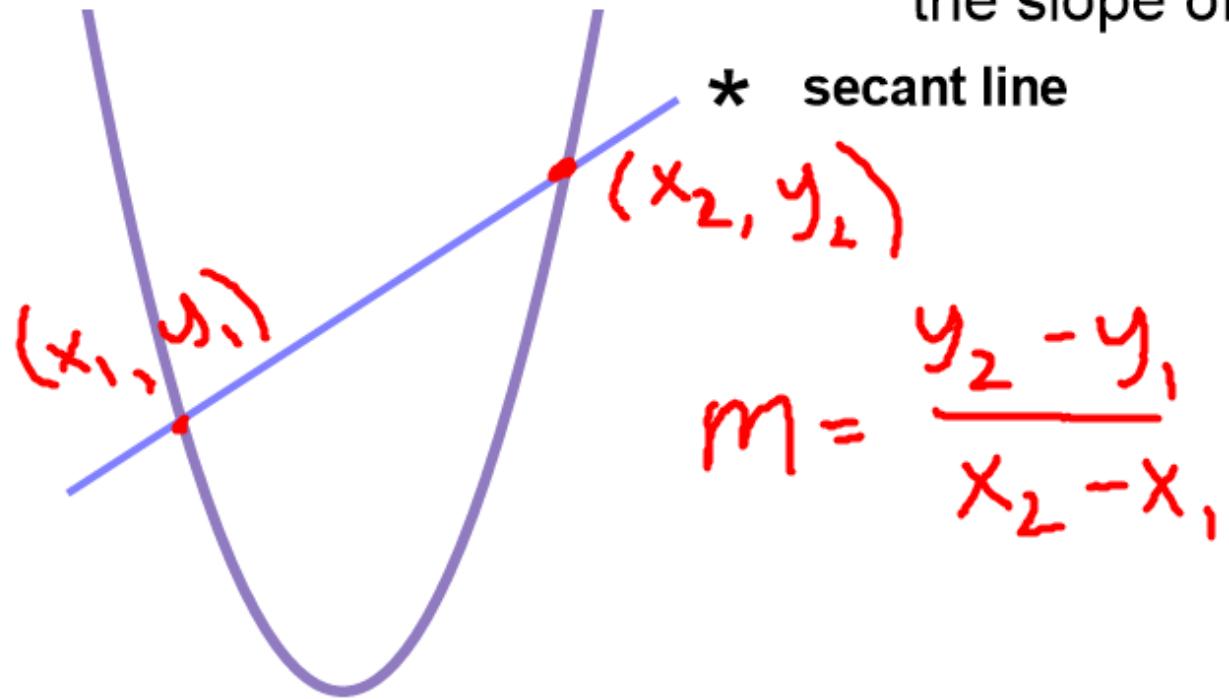
① Division: denominator  $\neq 0$

② Square Roots:  
expression under  $\sqrt{\phantom{x}} \geq 0$

③ Both: expression  $> 0$

# Average Rate of Change

- \* Average Rate of Change:  
the slope of the secant line



- \* *Average Rate of Change* = 
$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Ex 7: Find the average rate of change of the following function from  $-1$  to  $3$ .

$$f(x) = x^3 - 2x + 3 \quad \frac{y_2 - y_1}{x_2 - x_1}$$

$x$	$y$
$-1$	$4$
$3$	$24$

$$(-1)^3 - 2(-1) + 3$$

$$(3)^3 - 2(3) + 3$$

~~27~~

$$\frac{24 - 4}{3 - (-1)} = \frac{20}{4} = \boxed{5}$$

$$\text{Ex 8: } f(x) = \underline{x^2 - 2x}$$

$$\frac{y_2 - y_1}{x_2 - x},$$

a) Find the average rate of change from 2 to  $x$ .

$x_1 \quad x_2$

$$\begin{array}{c|c} x & y \\ \hline 2 & 0 \\ x & x^2 - 2x \end{array} \quad (2)^2 - 2(2)$$

$$\frac{5}{15} = \frac{5 \cdot 1}{5 \cdot 3} = \frac{1}{3}$$

$$\frac{(x^2 - 2x) - 0}{x - 2} = \frac{x^2 - 2x}{x - 2} = \frac{x(x-2)}{1(x-2)}$$

$$= \boxed{x}$$

Ex 8:  $f(x) = x^2 - 2x$

b) Use the previous result to find the average rate of change from 2 to 4. Interpret this result.

Average:  $x$   
↓  
4

Plug 4 in  
for  $x$ .

The slope of the secant  
line is 4.

$$\text{Ex 8: } f(x) = x^2 - 2x \quad y = mx + b$$

c) Find an equation of the secant line containing  $(2, f(2))$  and  $(4, f(4))$ .

$$\begin{array}{c|cc} x & \nearrow \\ \hline 2 & 0 \\ 4 & 8 & (4)^2 - 2(4) \end{array}$$

$$m = 4 \quad (\text{from part b})$$

$$\begin{aligned} 0 &= 4(2) + b \\ 0 &= 8 + b \\ b &= -8 \end{aligned} \quad \left. \begin{aligned} y &= 4x - 8 \\ &\star \end{aligned} \right\}$$

Ex 9:  $f(x) = 5x - 2$

a) Find the average rate of change from 1 to  $x$ .

$x$	$y$
1	3
$x$	$5x - 2$

$$5(1) - 2$$

$$\frac{(5x-2) - 3}{x-1} = \frac{5x-5}{x-1} = \frac{5(x-1)}{\cancel{(x-1)}} = \boxed{5}$$

Ex 9:  $f(x) = 5x - 2$

- b) Use the previous result to find the average rate of change from 1 to 3. Interpret this result.

The slope from part a  
was 5.

This slope stays the same.

5

The slope of the secant  
line is 5.

Ex 9:  $f(x) = 5x - 2$

c) Find an equation of the secant line containing  $(1, f(1))$  and  $(3, f(3))$ .

$$m = 5$$

x	y
1	$5(1) - 2$
3	$5(3) - 2$

$$3 = 5(1) + b$$

$$3 = 5 + b$$

$$b = -2$$

$$y = 5x - 2$$