

Lesson 9.4: Applications of Systems of Equations

The ⁺sum of two numbers is 7 and the difference of their squares is 21. Find the numbers.

$$\begin{cases} x + y = 7 \rightarrow x = \boxed{7 - y} \\ x^2 - y^2 = 21 \end{cases}$$

$$(7 - y)^2 - y^2 = 21$$

$$\cancel{y^2} - 14y + 49 - \cancel{y^2} = 21$$

$$-14y = -28$$

$$\boxed{y = 2 \quad x = 5}$$

The product of two numbers is 10 and the sum of their squares is 29. Find the numbers.

$$\begin{cases} xy = 10 \rightarrow y = \frac{10}{x} \\ x^2 + y^2 = 29 \end{cases}$$

$$x^2 + \left(\frac{10}{x}\right)^2 = 29$$
$$x^2 \left(x^2 + \frac{100}{x^2}\right) = (29)x^2$$

$$x^4 + 100 = 29x^2$$

$$x^4 - 29x^2 + 100 = 0$$

$$(x^2 - 25)(x^2 - 4)$$
$$\underbrace{(x+5)}_{\downarrow} \underbrace{(x-5)}_{\downarrow} \underbrace{(x+2)}_{\downarrow} \underbrace{(x-2)}_{\downarrow}$$

$$x = -5, 5, -2, 2$$

$$y = -2, 2, -5, 5$$

The difference of two numbers is the same as their product, and the sum of their reciprocals is 5. Find the numbers.

$$\begin{cases} x - y = xy \rightarrow x = y + xy \\ \frac{1}{x} + \frac{1}{y} = 5 \end{cases}$$
$$\frac{x}{1+x} = \frac{y(1+x)}{1+x}$$
$$y = \frac{x}{x+1}$$

$$\frac{1}{x} + \frac{x+1}{x} = 5$$

$$x \cdot \frac{x+2}{x} = 5 \cdot x$$

$$x+2 = 5x$$

$$2 = 4x \rightarrow x = \frac{1}{2}$$

$$y = \frac{\frac{1}{2}}{\frac{1}{2}+1} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$$

$$\frac{1}{2} \text{ and } \frac{1}{3}$$

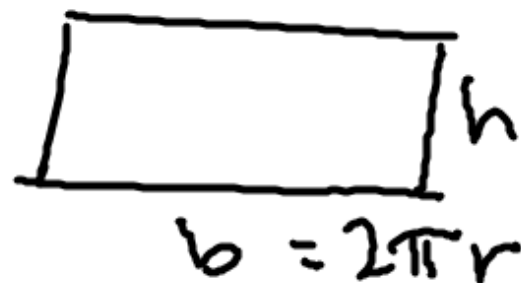
$$\left(\frac{1}{2}, \frac{1}{3} \right)$$

The perimeter of a rectangle is 36 inches and its area is 24 square inches.
What are its dimensions?



$$\begin{cases} 2b + 2h = 36 \\ bh = 24 \end{cases}$$

84 A rectangular piece of cardboard, whose area is 216 square centimeters, is made into a cylindrical tube by joining together two sides of the rectangle. If the tube is to have a volume of 224 cubic centimeters, what size cardboard should you start with?



$$\begin{aligned} \textcircled{1} \quad & bh = 216 \\ \textcircled{2} \quad & \pi r^2 \cdot h = 224 \end{aligned}$$

$$\begin{cases} 2\pi r h = 216 \\ \pi r^2 h = 224 \end{cases}$$

$$h = \frac{216}{2\pi r} = \frac{108}{\pi r}$$

$$\begin{aligned} \pi r^2 \left(\frac{108}{\pi r} \right) &= 224 \\ 108r &= 224 \\ r &= \frac{56}{27} \end{aligned}$$

$$h = \frac{108}{\pi \left(\frac{56}{27} \right)} \approx 16.57 \text{ cm}$$

$$b = 2\pi \left(\frac{56}{27} \right) \approx 13.03 \text{ cm}$$