Lesson 9.3: Solving Systems of Non-linear Equations

Substitution Method:

1. Take one of the two equations and solve for either x or y

2. Substitute into the other equation

3. Solve for the variable

4. Plug the answer in to any equation to find the other variable.

Elimination Method:

1. Multiply one or both equations so that the coefficients match
   (it's easiest if one of the coefficents is negative)

2. Add the equations

3. Solve for the remaining variable

4. Plug the answer in to any equation to find the other variable.

EX 1: $\begin{cases} y = x^2 + 1 \\ y = \boxed{3x + 1} \end{cases}$

$$3x + 1 = x^2 + 1$$

$$0 = x^2 - 3x$$

$$0 = x(x - 3)$$

$X = 0$

$y = 0^2 + 1$

$y = 1$

$(0, 1)$

$X = 3$

$y = 3^2 + 1$

$y = 10$

$(3, 10)$

EX 2: $\begin{cases} y = \sqrt{x} \\ y = 2 - x \end{cases}$

$(2-x)(2-x)$

$(\sqrt{x})^2 = (2-x)^2$

$x = x^2 - 4x + 4$

$0 = x^2 - 5x + 4$

$x^2 = 4$

$(x - 1)(x - 4)$

$x = 1$

$y = \sqrt{1} = 1$

$\boxed{(1, 1)}$

$x = 4$

$y = \sqrt{4} = 2$

$\cancel{(4, 2)}$

Extraneous

$y = 2 - 4$

$y = -2$

EX 3:

$$\begin{cases} x^2 + y^2 = 4 \\ -1(x^2 + 2x + y^2 = 0) \end{cases}$$

$$x^2 + y^2 = 4$$

$$+ \quad -x^2 - 2x - y^2 = 0$$

$$-2x = 4$$

$$\boxed{x = -2}$$

$y:$ $(-2)^2 + y^2 = 4$

$$4 + y^2 = 4$$

$$y^2 = 0$$

$$\boxed{y = 0}$$

$$\boxed{(-2, 0)}$$

EX 4: $\begin{cases} (y)^3 = (\sqrt[3]{x})^3 \rightarrow y^3 = x \\ y^2 = 2x \end{cases}$

$y^2 = 2y^3$

$0 = 2y^3 - y^2$

$0 = y^2(2y - 1)$

$y = 0 \qquad\qquad y = \frac{1}{2}$

$x = 0 \qquad\qquad x = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$\boxed{(0,0)} \qquad\qquad \boxed{\left(\frac{1}{8}, \frac{1}{2}\right)}$

$$\begin{cases} x^2 + y^2 = 10 \\ \dfrac{xy}{y} = \dfrac{3}{y} \end{cases} \to X = \dfrac{3}{y}$$

$$\left(\dfrac{3}{y}\right)^2 + y^2 = 10$$

$$\left(\dfrac{9}{y^2} + y^2 = 10\right) y^2$$

$$9 + y^4 = 10y^2$$

$$y^4 - 10y^2 + 9 = 0$$

$$(y^2 - 9)(y^2 - 1) = 0$$

$(y+3)(y-3)(y+1)(y-1)$

$y = -3, 3, -1, 1$

$X = -1, 1, -3, 3$

$(-1, -3)$

$(1, 3)$

$(-3, -1)$

$(3, 1)$

$$\begin{cases} \ln x = 4\ln y \\ \log_3 x = 2 + 2\log_3 y \end{cases}$$

$$e^{\ln x} = e^{\ln y^4}$$

$$x = y^4$$

Substitute:

$$\log_3 y^4 = 2 + 2\log_3 y$$

$$\log_3 y^4 = 2 + \log_3 y^2$$

$$-\log_3 y^2 \qquad\qquad -\log_3 y^2$$

$$\log_3 y^4 - \log_3 y^2 = 2$$

$$\log_3\left(\frac{y^4}{y^2}\right) = 2$$

$$3^{\log_3(y^2)} = 3^2$$

$$y^2 = 9$$

$$y = \pm 3$$

$$x = (3)^4 = 81$$

$$\boxed{(81, 3)}$$

$$4\log_3 y = 2 + 2\log_3 y$$

$$2\log_3 y = 2$$

$$\log_3 x$$

argument

$$\log_3 (27) = 3$$

$$\begin{cases} \log_x(2y) = 3 \\ \log_x(4y) = 2 \end{cases}$$

$$x^{\log_x(2y)} = x^3$$

$$\frac{2y}{2} = \frac{x^3}{2} \rightarrow y = \frac{x^3}{2}$$

Substitute

$$\log_x\left(4\left(\frac{x^3}{2}\right)\right) = 2$$

$$x^{\log_x(2x^3)} = x^2$$

$$2x^3 = x^2$$

$$0 = 2x^3 - x^2$$

$$x^2(2x-1) = 0$$

$$x \neq 0$$

extraneous

$$\boxed{x = \frac{1}{2}}$$

$$y = \frac{\left(\frac{1}{2}\right)^3}{2} = \frac{1}{16}$$

$$\boxed{\left(\frac{1}{2}, \frac{1}{16}\right)}$$

$$\begin{cases} x^2 - xy - 2y^2 = 0 \\ xy + x + 6 = 0 \end{cases}$$

$(x - 2y)(x + y)$

$-y(y) + (-y) + 6 = 0$

$x + y = 0$

$-y^2 - y + 6 = 0$

$x - 2y = 0$

$x = -y$

$-(y^2 + y - 6) = 0$

$x = 2y$

$-(y + 3)(y - 2) = 0$

$2y(y) + 2y + 6 = 0$

$y = -3$

$y = 2$

$2y^2 + 2y + 6 = 0$

$x = 3$

$x = -2$

$2(y^2 + y + 3) = 0$

$(3, -3)$

$(-2, 2)$

$y = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(3)}}{2(1)}$ → $\dfrac{-1 \pm \sqrt{-11}}{2}$ imaginary!

Circle all solutions to the following systems of equation.