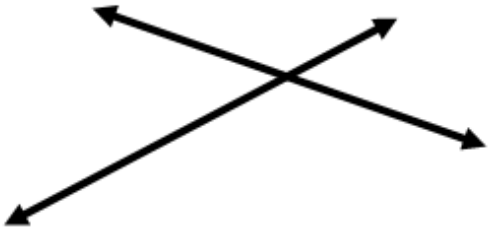
Lesson 9.2: Solving Systems of Equations and Inequalities

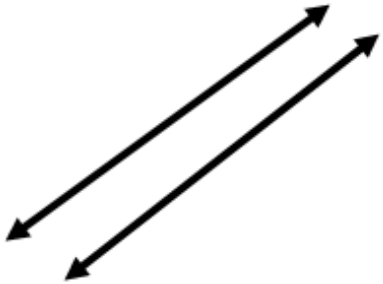
System of Equations: A collection of 2 or more equations with the same set of unknowns.

Example: $\begin{cases} 2x + y = -5 \\ 5x - 2y = -17 \end{cases}$

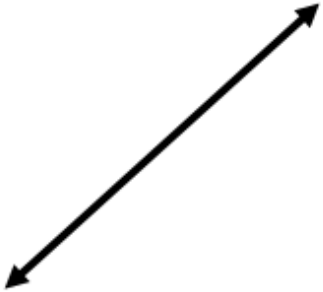
Solution of a systems of equations makes every equation true. Graphically, the solution is the point of intersection.



One Solution



No Solution
(Inconsistent)



Infinitely Many Solutions

Methods: Substitution, Elimination, Graphing

Substitution Method

$$\begin{cases} 2x + y = -5 \\ 5x - 2y = -17 \end{cases} \rightarrow$$

$y = -2x - 5$

(1) Solve for one of the variables:

$y = -2x - 5$

(2) Plug into other equation

$5x - 2(-2x - 5) = -17$

$5x + 4x + 10 = -17$

$9x = -27$

$x = -3$

(3) Find the other variable:

$y = -2(-3) - 5 = 1$

$(-3, 1)$

Substitution Method

$$\begin{cases} 2x + 3y = 7 \\ 12y = 28 - 8x \end{cases}$$

$2x + 3y = 7 \rightarrow 2x = -3y + 7 \rightarrow x = -\frac{3}{2}y + \frac{7}{2}$

$12y = 28 - 8\left(-\frac{3}{2}y + \frac{7}{2}\right)$

$12y = 28 + 12y - 28$

$12y = 12y$

$0 = 0$

Infinitely many

$\left(x, \frac{7}{3} - \frac{2}{3}x\right)$

$\frac{12y}{12} = \frac{28}{12} - \frac{8x}{12}$

$y = \frac{7}{3} - \frac{2}{3}x$

Elimination Method

$$3\begin{cases}(2x-5y=14)\\ -6x+15y=13\end{cases}$$

(1) Match coefficient

$$\begin{array}{r} 6x-15y=42 \\ +\ -6x+15y=13 \\ \hline 0 \neq 55 \end{array}$$

(2) Add / Subtract

No Solution

Elimination Method

$3(4x - 6y = 0)$

$2(10x + 9y = 8)$

① Match Coefficients

$12x - 18y = 0$

$+\ 20x + 18y = 16$

$32x = 16$

② Add

$32x = 16$

$x = \frac{1}{2}$

③ Find other variable.

$4(\frac{1}{2}) - 6y = 0$

$2 - 6y = 0$

$2 = 6y$

$y = \frac{1}{3}$

$(\frac{1}{2}, \frac{1}{3})$

Graphing Systems of Inequalities

$\leq, \geq$ = bold line

$<, >$ ⇒ dotted line

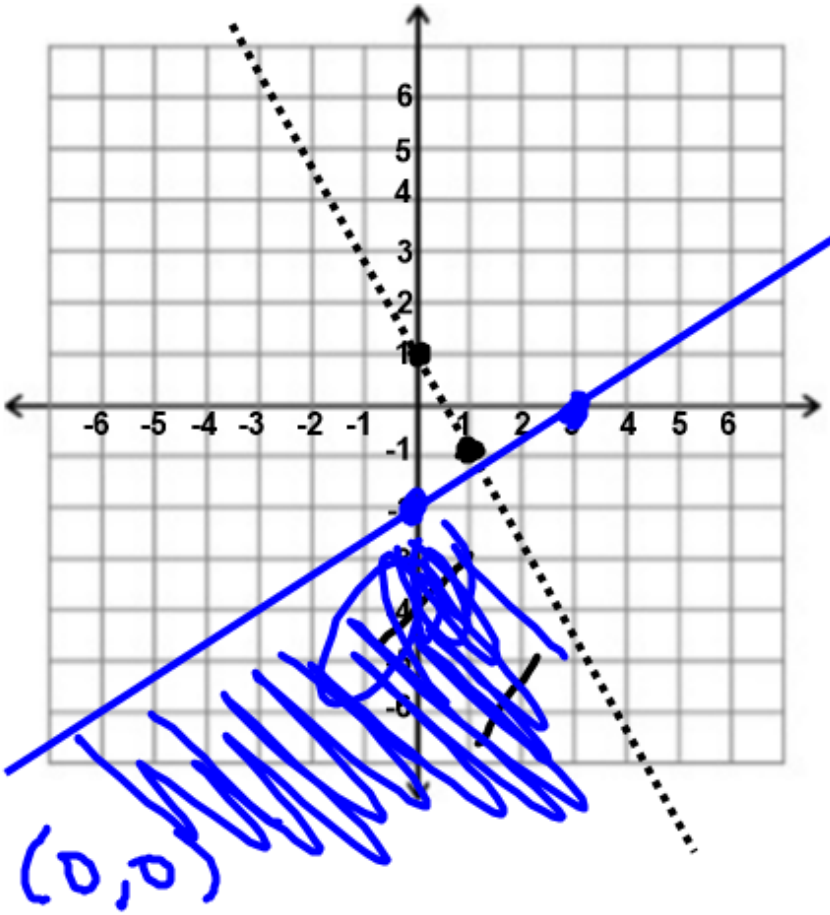
$$\begin{cases} 2x - 3y \geq 6 \\ 2x + y < 1 \end{cases}$$

↳ $y < -2x + 1$

Test: (0, 0)

$0 < -2(0) + 1$

$0 < 1$ ✓



(0, 0)

$0 \geq 6$ False

$2x - 3y \geq 6$

x	y
0	$-3y = 6 \rightarrow -2$
3	0

Write the systems of equations for the given context and then solve.

A caterer is making an ice cream punch by combining fruit juice and ice cream. The juice costs $2.25 per gallon and the ice cream costs $3.25 per gallon. She has 20 gallons of juice and needs to determine how much ice cream she should add so that the punch will cost $2.50 per gallon. How much ice cream should she add?

M = # of gallons of the mix

I = # of gallons ice cream

$$\begin{cases} M = 20 + I \\ 2.5 \cdot M = 2.25(20) + 3.25 \cdot I \end{cases}$$