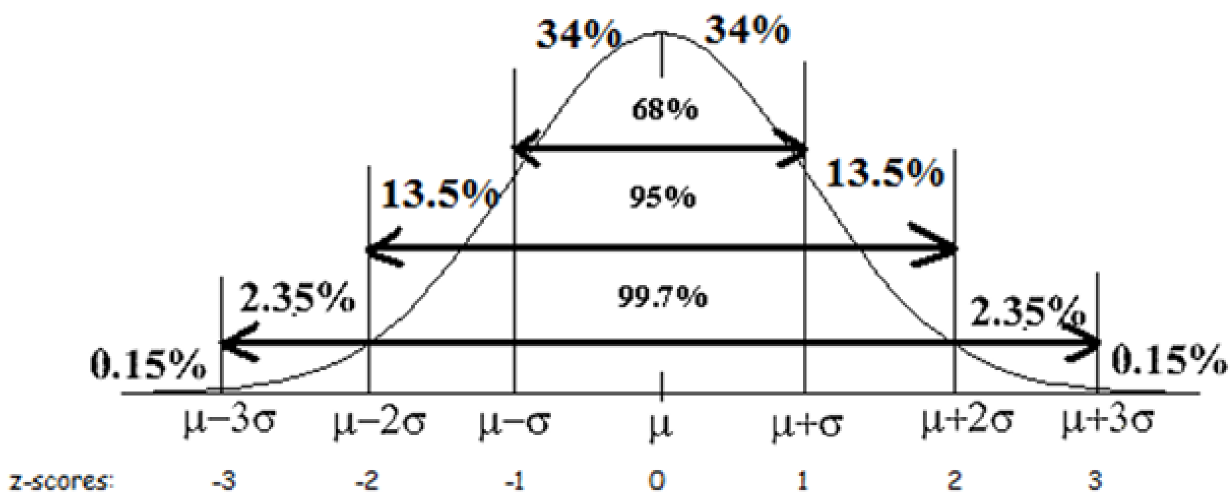
Lesson 8.3: Normal Distribution

The 68-95-99.7 Rule is a great way to approximate the probability of a normal distribution if you happen to be interested in a perfect number of standard deviations away from the mean. In most circumstances this is not the case. In order to deal with this situation we look at **z-scores** and use a **standard normal table** or **technology** to calculate probabilities.



The **standard Normal distribution** is the Normal distribution with **mean 0** and **standard deviation 1**. If a variable x has any Normal distribution $N(\mu, \sigma)$ with mean μ and standard deviation σ, then the **standardized variable** $\boxed{z = \frac{x-\mu}{\sigma}}$ has the standard Normal distribution $N(0, 1)$.

$$Z = \frac{x-\mu}{\sigma}$$

The **standard Normal table** is a table of **areas** under the standard Normal curve. The **table entry** for each value z is the **area** under the curve to the **left** of z.

The mean height of 18-24-year-old males in the United States is about 70.1 inches with a standard deviation of 2.7 inches. The mean height of 18-24-year-old females is about 64.8 inches, with a standard deviation of 2.5 inches.

a- What percentage of men are under 6 feet tall? Find the appropriate z-score to answer this question as well as the percentage requested.

$$z = \frac{x - \mu}{\sigma} = \frac{(72 - 70.1)}{2.7} \approx 0.70$$

$\mu = 70.1$

$\sigma = 2.7$

$x = 6(12) = 72$

From Table

.7580

$\boxed{75.80\%}$

The mean height of 18-24-year-old males in the United States is about 70.1 inches with a standard deviation of 2.7 inches. The mean height of 18-24-year-old females is about 64.8 inches, with a standard deviation of 2.5 inches.

b- What percentage of women are under 5 feet tall? Find the appropriate z-score to answer this question as well as the percentage requested.

$$z = \frac{x - \mu}{\sigma}$$

$\mu = 64.8$

$\sigma = 2.5$

$x = 5(12) = 60$

$$z = \frac{(60 - 64.8)}{2.5} \approx -1.92$$

Go To Table

.0274

$\downarrow$

$\boxed{2.74\%}$

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c- Derrick Rose became the youngest NBA player to receive the MVP award at age 22. His height is 6'3". What percent of men are taller than him? Find the appropriate z-score to answer this question as well as the percentage requested.

$$z = \frac{75 - 70.1}{2.7} \approx 1.81 \rightarrow$$

Go To Table

.9649

96.49%

(shorter)

$$x = 6(12) + 3 = 75$$

$$100 - 96.49 \approx \boxed{3.51\%}$$

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d- To work as a flight attendant for United Airlines, you must be between 5'2" and 6' tall. What percent of men of this age meet the height requirement? Find the appropriate z-score to answer this question as well as the percentage requested.

6' Tall

$Z = 0.70$

(Refer to c)

75.8%

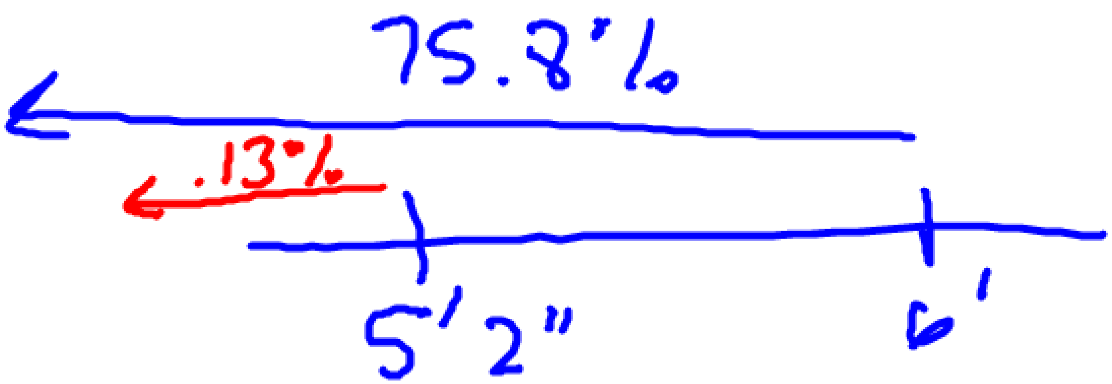
5' 2"

$$Z = \frac{62 - 70.1}{2.7} \approx -3$$

.0013

↓

.13%



$75.8 - .13$

$= \boxed{75.67\%}$

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e- What percent of women of this age do not meet the height requirement mentioned in part (a)? Find the appropriate z-score to answer this question as well as the percentage requested.

5'2"

$$z = \frac{62 - 64.8}{2.5} = -1.12$$

$\downarrow$

13.14%

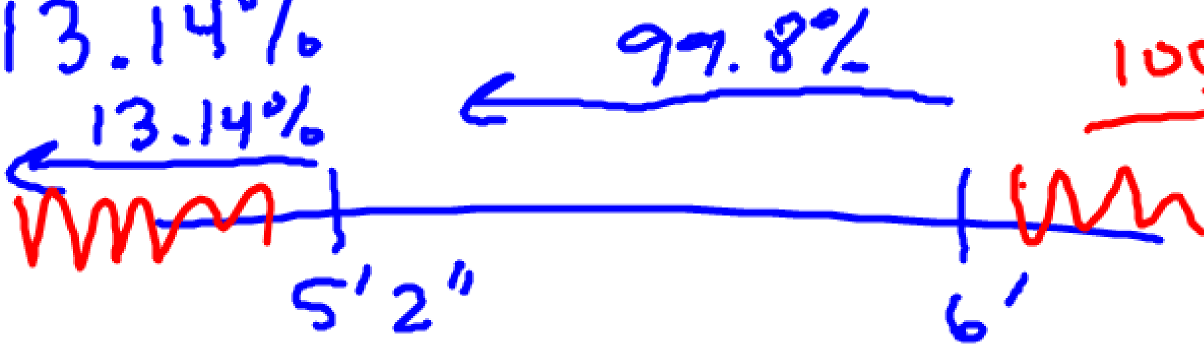
6'

$$z = \frac{72 - 64.8}{2.5} = 2.88$$

$\downarrow$

99.8%

100 − 99.8 = .2%



$$13.14 + 0.2 = \boxed{13.34\%}$$

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f- How tall should a woman be to be in the top 10%?

$$z = 1.28$$

$$z = \frac{x - \mu}{\sigma}$$

$$2.5 \cdot 1.28 = \frac{x - 64.8}{2.5} \cdot 2.5$$

$$3.2 = x - 64.8$$

$$+64.8 \qquad +64.8$$

$$x = 68''$$

$$\boxed{\text{Over } 68'' \text{ or } 5'\,8''}$$

At some fast-food restaurants, customers who want a lid for their drinks get them from a large stack left near straws, napkins, and condiments. The lids are made with a small amount of flexibility so they can be stretched across the mouth of the cup and snugly secured. When lids are too small or too large, customers can get very frustrated especially if they end up spilling their drinks. At one particular restaurant, large drink cups require lids with a "diameter" of between 3.95 and 4.05 inches. The restaurant's lid supplier claims that the diameter of their large lids follows a Normal distribution with mean 3.98 inches and standard deviation 0.02 inches. Assume that the supplier's claim is true.

a- What percent of large lids are too small to fit? Find the appropriate z-score to answer this question as well as the percentage requested.

$z = \frac{3.95 - 3.98}{.02} = -1.50 \rightarrow$ Table .0668

$\boxed{6.68\%}$

$x = 3.95$

$\mu = 3.98$

$\sigma = .02$

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b- What percent of large lids are too big to fit? Find the appropriate z-score to answer this question as well as the percentage requested.

$$z = \frac{4.05 - 3.98}{.02} = 3.5\%$$

$\downarrow$ Not on Chart (too big)

$\downarrow$

$\boxed{0\%}$

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c- 25% of the lids are smaller than what value?

.02

$$-0.67 = \frac{x - 3.98}{.02} \cdot .02$$

$$\boxed{x = 3.967}$$