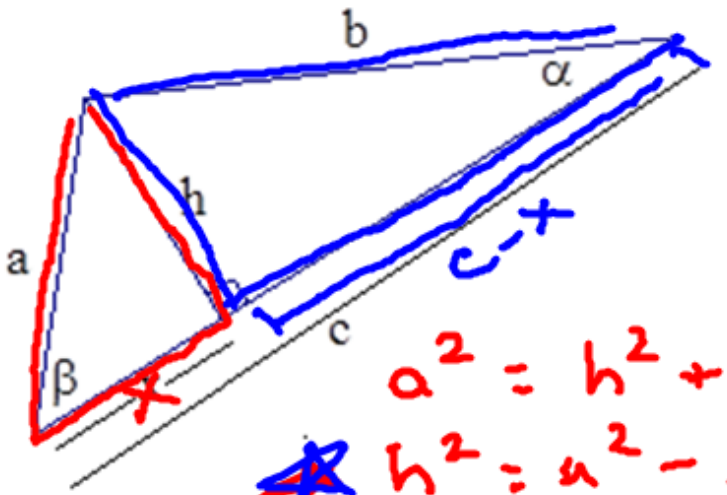
Lesson 7.5: Law of Cosines



$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$a^2 = h^2 + x^2$

$h^2 = a^2 - x^2$ ✓

$\cos\beta = \frac{x}{a}$

$x = a\cos\beta$ ✓

$b^2 = h^2 + (c-x)^2$

$b^2 = h^2 + c^2 - 2cx + x^2$

$b^2 = a^2 - x^2 + c^2 - 2cx + x^2$

$b^2 = a^2 + c^2 - 2cx$

$$\boxed{b^2 = a^2 + c^2 - 2ac\cos\beta}$$ ✓

Solve the triangle. SAS

$b = 4, c = 6, \alpha = 44°$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Find a:

$\sqrt{a^2} = \sqrt{4^2 + 6^2 - 2\cdot 4\cdot 6\cdot\cos(44°)}$

$\boxed{a \approx 4.18}$

$\gamma = 180 - 44 - 41.7$

$\boxed{\gamma = 94.3°}$

Find β:

$4^2 = 4.18^2 + 6^2 - 2\cdot 4.18\cdot 6\cos\beta$

$4^2 - 4.18^2 - 6^2 = -2\cdot 4.18\cdot 6\cos\beta$

$-37.4724 = -50.16\cos\beta$

$\cos\beta = \frac{-37.4724}{-50.16} \rightarrow \beta = \cos^{-1}\left(\frac{-37.4724}{-50.16}\right)$

$\boxed{\beta = 41.7°}$

Solve the triangle.

$a = 9, b = 4, c = 6$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Find α:

$$9^2 = 4^2 + 6^2 - 2\cdot 4\cdot 6\cos\alpha$$

$$9^2 - 4^2 - 6^2 = -2\cdot 4\cdot 6\cos\alpha$$

$$29 = -48\cos\alpha$$

$$\alpha = \cos^{-1}\left(\frac{29}{-48}\right) \approx \boxed{127.2^\circ}$$

Find β:

$$4^2 = 9^2 + 6^2 - 2\cdot 9\cdot 6\cos\beta$$

$$-101 = -108\cos\beta$$

$$\beta = \cos^{-1}\left(\frac{-101}{-108}\right) \approx \boxed{20.7^\circ}$$

$$\boxed{\gamma = 32.1^\circ}$$

A motorized sailboat leaves Naples, Florida, bound for Key West, 150 miles away. Maintaining a constant speed of 15 miles per hour, but encountering heavy crosswinds and strong currents, the crew finds that after 4 hours, that the sailboat is off course by 20°. How far is the sailboat from Key West at this time?

$$\sqrt{x^2} = \sqrt{60^2 + 150^2 - 2 \cdot 60 \cdot 150 \cos(20^\circ)}$$

$$x \approx \boxed{95.8 \text{ miles}}$$