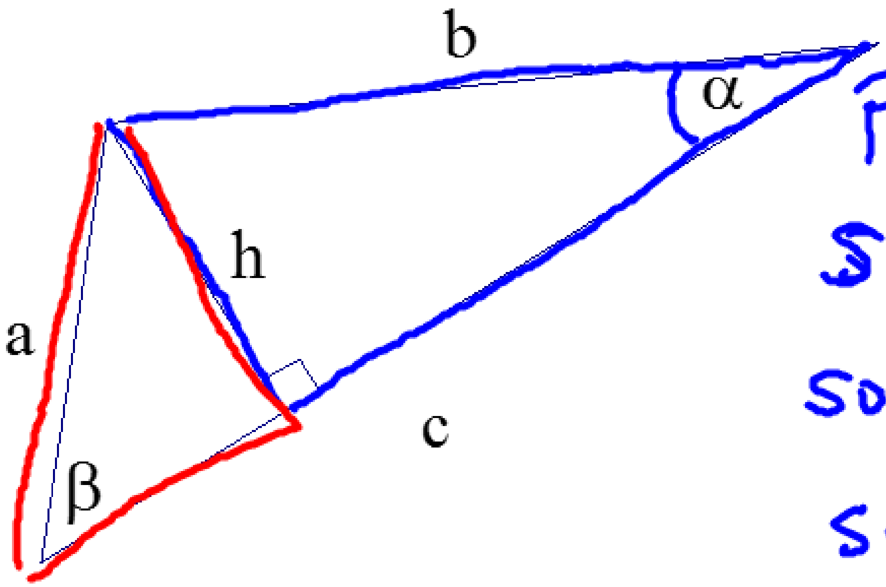
Lesson 7.4: Law of Sines

$$\boxed{\underbrace{\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}} = \frac{\sin\gamma}{c}}$$



Proof:

$\sin\alpha = \frac{h}{b}$ $\sin\beta = \frac{h}{a}$

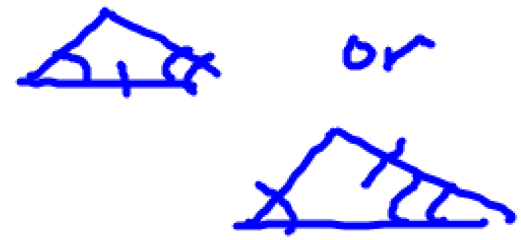
so, $h = b\sin\alpha$

$\sin\beta = \frac{b\sin\alpha}{a}$

Divide by b.

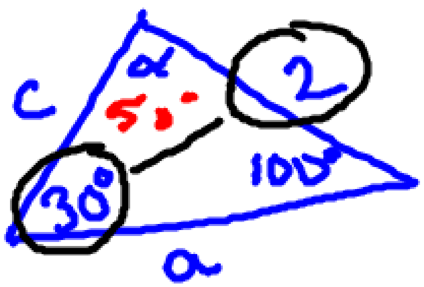
$\frac{\sin\beta}{b} = \frac{\sin\alpha}{a}$. ✓

With the Law of Sines, we can solve any triangle with 2 angles and a side (ASA or AAS)



Solve the Triangle.

$$\beta = 30°, \gamma = 100°, b = 2$$



$$\alpha = 180° - 100° - 30° = \boxed{50°}$$

$$\frac{\sin 30°}{2} = \frac{\sin 100°}{c}$$

$$\frac{2 \sin 100°}{\sin 30°} = \frac{c \sin 30°}{\sin 30°}$$

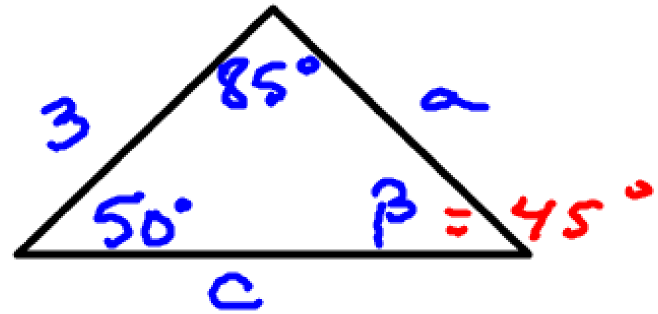
$$c = \frac{2 \sin 100°}{\sin 30°} \approx \boxed{3.9}$$

$$\frac{\sin 30°}{2} = \frac{\sin(50°)}{a}$$

$$a = \frac{2 \sin 50°}{\sin 30°} \approx \boxed{3.1}$$

Solve the Triangle.

$$\alpha = 50°, \gamma = 85°, b = 3$$



$$\beta = 180 - 85 - 50 = \boxed{45°}$$

Side a:

$$\boxed{\frac{\sin 45°}{3}} = \frac{\sin 50°}{a} \rightarrow a = \frac{3 \sin 50}{\sin 45} \approx \boxed{3.3}$$

Side c:

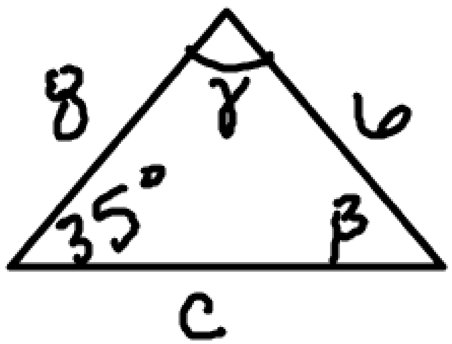
$$\frac{\sin 45°}{3} = \frac{\sin 85°}{b}$$

$$\boxed{b \approx 4.2}$$

We can also solve SSA triangles with the Law of Sines. This creates the ambiguous case (since SSA triangles can have zero, one or two solutions).

Solve the Triangle.

$$\alpha = 35°, a = 6, b = 8$$



* $\beta_2 = 180° - \beta_1$ $\quad \gamma_2 = 180 - \gamma_1$

$\alpha_2 = 180° - \alpha_1$

$\underline{\beta}$

$$\frac{\sin 35°}{6} = \frac{\sin\beta}{8}$$

$$\sin\beta = \frac{8\sin 35}{6}$$

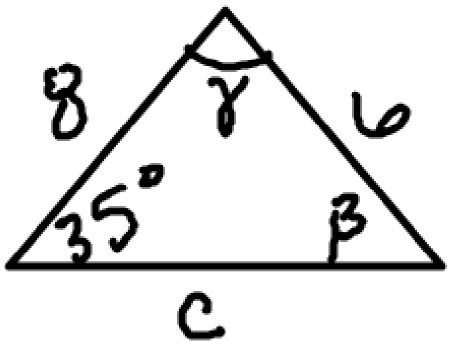
$$\beta_1 = \sin^{-1}\left(\frac{8\sin 35°}{6}\right) \approx \boxed{49.9°}$$

$$\beta_2 = 180° - 49.9° = \boxed{130.1°}$$

* $If\ \alpha + \beta_2 < 180°, then\ there\ is\ a\ 2nd\ triangle.$

Given $\quad$ secondary

We have a 2nd Triangle!



Triangle 1:

$$\boxed{\beta_1 = 49.9^\circ}$$

$$\gamma_1 = 180 - 49.9 - 35$$

$$\boxed{\gamma_1 = 95.1^\circ}$$

$$\frac{\sin 35}{6} = \frac{\sin 95.1^\circ}{c_1}$$

$$c_1 = \frac{6 \sin 95.1^\circ}{\sin 35} \approx \boxed{10.4}$$

Triangle 2:

$$\boxed{\beta_2 = 130.1^\circ}$$

$$\gamma_2 = 180 - 130.1 - 35$$

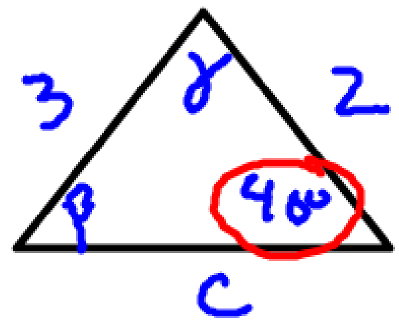
$$\boxed{\gamma_2 = 14.9^\circ}$$

$$\frac{\sin 35}{6} = \frac{\sin 14.9}{c_2}$$

$$c_2 = \frac{6 \sin 14.9}{\sin 35} \approx \boxed{2.7}$$

Solve the Triangle.

$\alpha = 40°, a = 3, b = 2$



Ambiguous!

$\underline{\beta}$

$\frac{\sin 40}{3} = \frac{\sin \beta}{2}$

$\sin\beta = \frac{2\sin 40}{3} \rightarrow \beta_1 = \sin^{-1}\left(\frac{2\sin 40°}{3}\right) \approx 25.4°$

$\beta_2 = 180 - 25.4° = \underline{154.6°}$

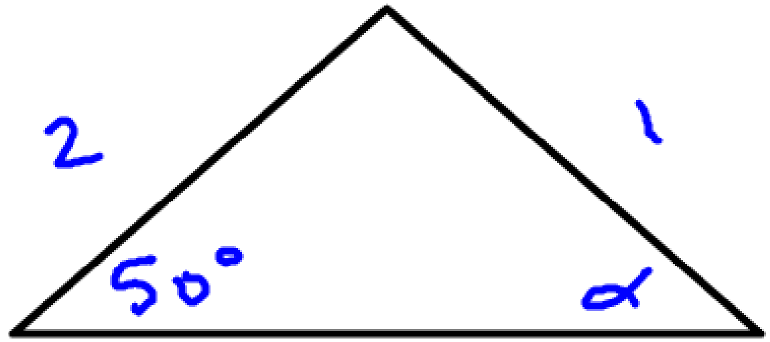
$40° + 154.6° = \cancel{194.6°}$ 1 Triangle

$\gamma_1 = 180 - 40 - 25.4° = \boxed{114.6°}$

$\frac{\sin 40}{3} = \frac{\sin 114.6}{c} \rightarrow c = \frac{3\sin 114.6}{\sin 40} \approx \boxed{4.2}$

Solve the Triangle.

$\gamma = 50°, a = 2, c = 1$



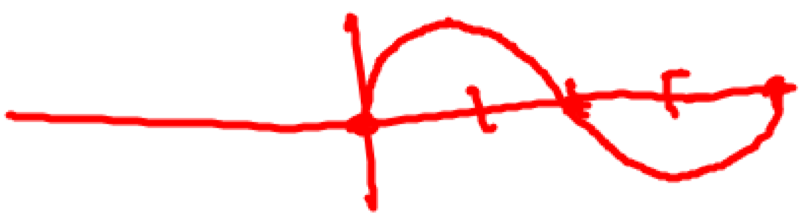
α:

$$\frac{\sin 50°}{1} = \frac{\sin \alpha}{2}$$

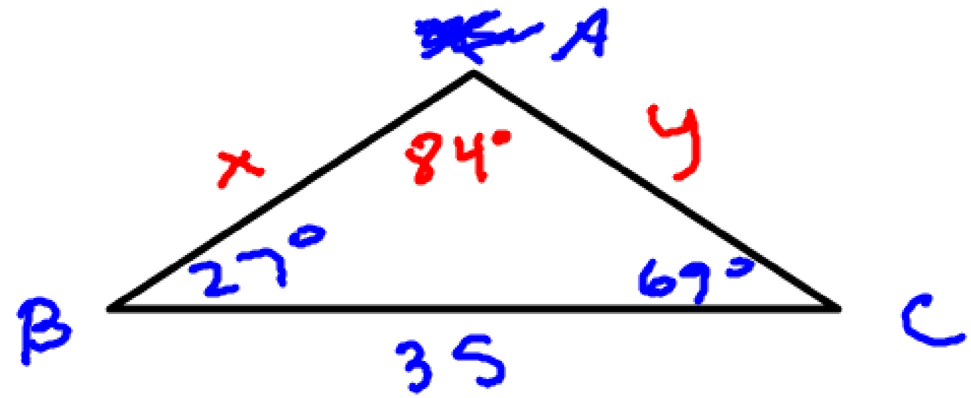
$$\sin \alpha = \frac{2 \sin 50°}{1}$$

Error

$$\alpha = \sin^{-1}(\underbrace{2\sin 50°}_{1.5}) \rightarrow$$ No Triangle



An airplane is flying between two airports that are 35 mi apart. The radar in one airport registers a 27° angle between the horizontal and the airplane. The radar system in the other airport registers a 69° angle between the horizontal and the airplane. How far is the airplane from each airport to the nearest tenth of a mile?



x:

$$\frac{\sin 84}{35} = \frac{\sin 69}{x}$$

$$\boxed{x \approx 32.9 \text{ miles}}$$

y:

$$\frac{\sin 84}{35} = \frac{\sin 27}{y}$$

$$\boxed{y \approx 16.0 \text{ miles}}$$