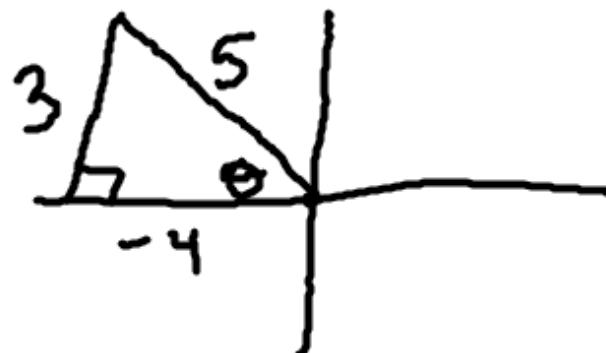


Lesson 7.3: Trig Identities

Use the information given about the angle to find the exact angle value for sine, cosine & tangent ratios.

1- $\sin \theta = \frac{3}{5}$, $\frac{\pi}{2} < \theta < \pi$



$$\sin = \frac{\text{opp}}{\text{hyp}}$$

$$\cos \theta = \frac{\text{adj.}}{\text{hyp.}} = -\frac{4}{5}$$

$$\tan \theta = \frac{\text{opp.}}{\text{adj.}} = -\frac{3}{4}$$

$$a^2 + b^2 = c^2$$

$$3^2 + b^2 = 5^2$$

$$b^2 = 25 - 9$$

$$b^2 = 16$$

$$b =$$

Use the information found in question 1 about the angle to find the exact value of the following functions.

$$6- \cos(2\theta)$$

$$\sin \theta = \frac{3}{5}, \cos \theta = -\frac{4}{5}; \tan \theta = -\frac{3}{4}$$

$$* \cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= \left(-\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25}$$

$$= \boxed{\frac{7}{25}}$$

Use the information found in question 1 about the angle to find the exact value of the following functions.

$$8- \sin(3\theta)$$

$$\sin \theta = \frac{3}{5}, \cos \theta = -\frac{4}{5}; \tan \theta = -\frac{3}{4}$$

$$\sin(2\theta + \theta)$$

$$* \sin(\alpha \pm \beta) = \underline{\sin \alpha \cos \beta} \pm \underline{\cos \alpha \sin \beta}$$

$$\begin{array}{c} \cancel{\sin(2\theta)} \cdot \cancel{\cos \theta} + \cancel{\cos 2\theta} \cdot \cancel{\sin \theta} \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ * \cancel{\frac{-24}{25}} \cdot -\frac{4}{5} + \frac{7}{25} \cdot \frac{3}{5} \end{array}$$

$$\frac{+96}{125} + \frac{21}{125} = \boxed{\frac{117}{125}}$$

Use the information found in question 1 about the angle to find the exact value of the following functions.

$$11 - \sin\left(\frac{\theta}{2}\right)$$



$$\sin \theta = \frac{3}{5}, \cos \theta = -\frac{4}{5}; \tan \theta = -\frac{3}{4}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$= \pm \sqrt{\frac{1 - (-\frac{4}{5})}{2}}$$

$$= \sqrt{\frac{1 + \frac{4}{5}}{2}}$$

$$= \sqrt{\frac{\frac{9}{5} + \frac{4}{5}}{2}}$$

$$= \sqrt{\frac{9/5}{2}}$$

$$= \sqrt{\frac{9}{10}}$$

$$= \frac{3}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}}$$

$$= \boxed{\frac{3\sqrt{10}}{10}}$$

Find exact values. #'s 13 - 20

$$14- \cos\left(-\frac{7\pi}{12}\right) = \cos\left(-\frac{4\pi}{12} + -\frac{3\pi}{12}\right)$$

sum/difference

$$= \cos\left(-\frac{\pi}{3} + -\frac{\pi}{4}\right)$$

$$\ast \cos(\alpha + \beta) = \underline{\cos \alpha \cos \beta + \sin \alpha \sin \beta}$$

$$= \underline{\cos\left(-\frac{\pi}{3}\right)} \underline{\cos\left(-\frac{\pi}{4}\right)} - \underline{\sin\left(-\frac{\pi}{3}\right)} \underline{\sin\left(-\frac{\pi}{4}\right)}$$

$$\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{3}}{2}\right) \left(-\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$$

Find exact values.

$$19 - \sin\left(-\frac{\pi}{8}\right)$$

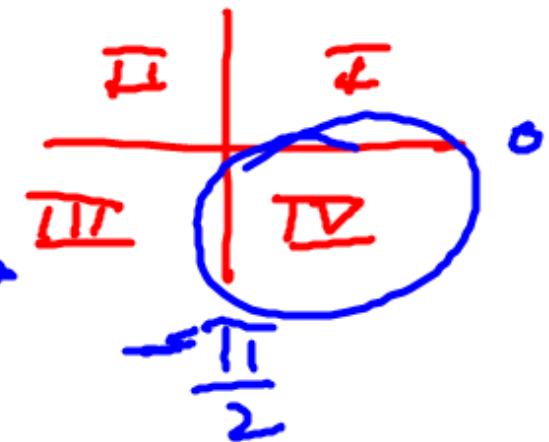
half-angle

$$\sin\left(\frac{1}{2}(-\frac{\pi}{8})\right)$$

Formula

$$\sin\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\sin\left(\frac{1}{2}\theta\right)$$



$$= -\sqrt{\frac{1 - \cos(-\frac{\pi}{8})}{2}}$$

$$= -\sqrt{\frac{\left(1 - \frac{\sqrt{2}}{2}\right) \cdot 2}{2}}$$

$$= -\sqrt{\frac{2 - \sqrt{2}}{4}}$$

$$= \boxed{-\frac{\sqrt{2 - \sqrt{2}}}{2}}$$

Establish the following identity.

22- $\cos \theta (\tan \theta + \cot \theta) = \csc \theta$

$$\cos \theta (\tan \theta + \cot \theta)$$

$$= \cos \theta \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right)$$

$$= \cos \theta \left(\frac{\sin \theta}{\cos \theta} \cdot \frac{\sin \theta}{\sin \theta} + \frac{\cos \theta}{\sin \theta} \cdot \frac{\cos \theta}{\cos \theta} \right)$$

$$= \cos \theta \left(\frac{\sin^2 \theta}{\sin \theta \cos \theta} + \frac{\cos^2 \theta}{\sin \theta \cos \theta} \right)$$

$$= \cos \theta \left(\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} \right)$$

$$= \cos \theta \left(\frac{1}{\sin \theta \cos \theta} \right)$$

$$= \frac{1}{\sin \theta}$$

$$= \csc \theta \quad \checkmark$$

$$* \quad \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$* \quad \cos^2 \theta + \sin^2 \theta = 1$$

$$* \quad \csc \theta = \frac{1}{\sin \theta}$$

Establish the following identity.

$$* \sec \theta = \frac{1}{\cos \theta}$$

$$27- \underline{\sec(2\theta)} = \boxed{\frac{\sec^2 \theta}{2 - \sec^2 \theta}}$$

$$\frac{\sec^2 \theta}{2 - \sec^2 \theta} = \frac{\frac{1}{\cos^2 \theta}}{2 - \frac{1}{\cos^2 \theta}} = \frac{\frac{1}{\cos^2 \theta}}{\frac{2\cos^2 \theta}{\cos^2 \theta} - \frac{1}{\cos^2 \theta}}$$

$$\therefore \frac{\frac{1}{\cos^2 \theta}}{\frac{2\cos^2 \theta - 1}{\cos^2 \theta}} = \frac{1}{\cos^2 \theta} \cdot \frac{\cos^2 \theta}{2\cos^2 \theta - 1}$$

$$\frac{1}{\cos 2\theta} \\ \frac{1}{\cos^2 \theta - \sin^2 \theta}$$

$$\therefore \frac{1}{2\cos^2 \theta - 1} = \frac{1}{2\cos^2 \theta - (\cos^2 \theta + \sin^2 \theta)}$$

$$* \boxed{\cos^2 \theta + \sin^2 \theta =}$$

Not Finished! We will finish next time in class!