Lesson 7.2: Solving Trig Equations

1. Isolate the Trig Function
2. Apply the Inverse and Find the Primary Angle
3. Find the Secondary Angle

Cos/Sec	Sin/Csc	Tan/Cot
Opposite of the Primary Angle	$\pi - Primary\ Angle$	No Secondary Angle

4. Solve for θ.
5. Write the General Equation

Cos/Sec/Sin/Csc	Tan/Cot
$+\frac{2\pi}{c}k$	$+\frac{\pi}{c}k$

6. Find the requested solutions

Solve and write the general equation(s).

$$2\cos 3\theta + 5 = 4$$

1. $-5 \quad -5$

$$\frac{2}{2}\cos 3\theta = -\frac{1}{2}$$

$$\cos 3\theta = -\frac{1}{2}$$

2. $3\theta = \cos^{-1}\left(-\frac{1}{2}\right)$

3. $\frac{3\theta_1}{3} = \frac{2\pi}{3} \cdot \frac{1}{3}$ $\qquad$ $\frac{3\theta_2}{3} = -\frac{2\pi}{3} \cdot \frac{1}{3}$

4. $\theta_1 = \frac{2\pi}{9}$ $\qquad$ $\theta_2 = -\frac{2\pi}{9}$

5. $$\boxed{\begin{aligned} \theta_1 &= \frac{2\pi}{9} + \frac{2\pi}{3} \cdot K \\ \theta_2 &= -\frac{2\pi}{9} + \frac{2\pi}{3} \cdot K \end{aligned}}$$

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6. Find the requested solutions

K helps find specific values.
K is an integer

Solve each equation over the interval: $0 \le \theta < 2\pi$

$$3\tan\theta + 2\sqrt{3} = 3\sqrt{3}$$

$$-2\sqrt{3} \quad -2\sqrt{3}$$

$$\frac{3\tan\theta}{3} = \frac{\sqrt{3}}{3}$$

$$\tan\theta = \frac{\sqrt{3}}{3}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$$

$$\theta_1 = \frac{\pi}{6}$$

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6. Find the requested solutions

No Secondary

Equation: $\theta = \frac{\pi}{6} + \pi k$

Step 6:

$$\boxed{\frac{\pi}{6}, \frac{7\pi}{6}}$$

$$\frac{\pi}{6} + \pi(1) = \frac{\pi}{6} + \frac{6\pi}{6} = \frac{7\pi}{6}$$

$$\frac{\pi}{6} + \pi(2) = \frac{\pi}{6} + \frac{12\pi}{6} = \cancel{\frac{13\pi}{6}}$$

Too big

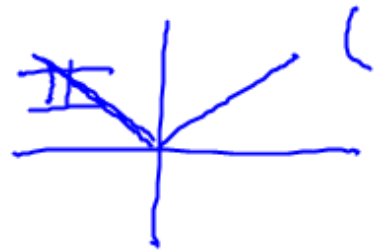
Solve each equation over the interval: $0 \le \theta < 2\pi$

$$4\sec^2\theta + 3 = 11$$
$$-3 \quad -3$$

$$\frac{4\sec^2\theta}{4} = \frac{8}{4}$$

$$\sqrt{\sec^2\theta} = \sqrt{2}$$

$$\sec\theta = \pm\sqrt{2}$$



$\sec\theta = \sqrt{2}$

$\theta = \sec^{-1}\sqrt{2}$

$\theta_1 = \frac{\pi}{4}$ $\quad \theta_2 = -\frac{\pi}{4}$

$\sec\theta = -\sqrt{2}$

$\theta = \sec^{-1}(-\sqrt{2})$

$\theta_3 = \frac{3\pi}{4}$ $\quad \theta_4 = -\frac{3\pi}{4}$

Equations: $\theta_1 = \frac{\pi}{4} + 2\pi K$

$\theta_2 = -\frac{\pi}{4} + 2\pi K$

$\theta_3 = \frac{3\pi}{4} + 2\pi K$

$\theta_4 = -\frac{3\pi}{4} + 2\pi K$

$-\frac{\pi}{4} + 2\pi(1) = \frac{7\pi}{4}$

$-\frac{3\pi}{4} + 2\pi(1)$

$$\boxed{\frac{\pi}{4}, \frac{7\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}}$$

Solve each equation and list 6 solutions.

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6. Find the requested solutions

$$3\csc 2\theta - 1 = -7$$
$$+1 \quad +1$$
$$\frac{3\csc 2\theta}{3} = \frac{-6}{3}$$
$$\csc 2\theta = -2$$
$$2\theta = \csc^{-1}(-2)$$

$$\frac{2\theta_1}{2} = -\frac{\pi}{6} \cdot \frac{1}{2}$$
$$\theta_1 = -\frac{\pi}{12}$$
$$\theta_1 = -\frac{\pi}{12} + \pi k$$
$$\theta_2 = \frac{7\pi}{12} + \pi k$$

$$\frac{2\theta_2}{2} = \pi - \left(-\frac{\pi}{6}\right) = \frac{7\pi}{6} \cdot \frac{1}{2}$$
$$\theta_2 = \frac{7\pi}{12}$$

$$-\frac{\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}, \frac{31\pi}{12}$$

$$-\frac{\pi}{12} + \pi$$

$$-\frac{\pi}{12} + \frac{12\pi}{12}$$

Solve each equation and list solutions.

$$\sin\left(\frac{\theta}{2}+\frac{\pi}{6}\right)=1$$

$\sin\theta = y$

$$\frac{\theta}{2}+\frac{\pi}{6}=\sin^{-1}(1)$$

$$\frac{\theta}{2}+\frac{\pi}{6}=\frac{\pi}{2} \quad -\frac{\pi}{6}$$

$$-\frac{\pi}{6}$$

$$2\cdot\frac{\theta}{2}=\frac{\pi}{3}\cdot 2$$

$$\theta=\frac{2\pi}{3}+\frac{2\pi}{\pi/2}k$$

$$\theta=\frac{2\pi}{3}+4\pi k$$

Solutions

$k=0$	$\frac{2\pi}{3}$
$k=1$	$14\pi/3$
$k=2$	$26\pi/3$
$k=3$	$38\pi/3$
$k=-1$	$-10\pi/3$