5.6: Applications of Logarithms

Compound Interest Formula

The amount A after t years due to a principal P invested at an annual interest rate r (in decimal form) compounded n times per year is:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Continuous Compound Interest

The amount A after t years due to a principal P invested at an annual interest rate r (in decimal form) compounded continuously is:

$$A = Pe^{rt}$$

Uninhibited Growth (k > 0)

Many natural phenomena have been found to follow the law that an amount *A* varies with time *t* according to:

$$A = A_0 e^{kt}$$

Uninhibited Decay (k < 0)

Many natural phenomena have been found to follow the law that an amount *A* varies with time *t* according to:

$$A = A_0 e^{kt}$$

Newton's Law of Cooling

The temperature *U* of a heated object at a given time *t* can be modeled by the following function:

$$U = T + (U_0 - T)e^{kt}$$

Where *T* is the constant temperature of the surrounding medium, U_0 is the initial temperature of the heated object, and *k* is a constant representing the rate of cooling

Logistic Model

In a logistic growth model, the population P after time t obeys the equation

$$P = \frac{c}{1 + ae^{kt}}$$

The model is a growth model if $k > 0$ and decay model if $k < 0$. The number c is often called the carrying capacity.

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How many years will it take for an initial investment of \$25,000 to grow to \$80,000? Assume a rate of interest of 7% compounded continuously.

$A = Pe^{rt}$

$A = 80{,}000$

$P = 25{,}000$

$r = 0.07$

$t = ?$

$$\frac{80{,}000}{25{,}000} = \frac{25{,}000}{25{,}000} e^{0.07t}$$

$$\ln 3.2 = \ln e^{0.07t}$$

$$\ln(3.2) = 0.07t$$

$$t = \frac{\ln(3.2)}{0.07} \approx \boxed{16.6 \text{ years}}$$

Page 323 #38

Sears charges 1.25% per month on the unpaid balance for customers with charge accounts (interest is compounded monthly). A customer charges $200 and does not pay her bill for 6 months. What is the bill at that time?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = ?$

$P = 200$

$r = 0.0125$

$n = 12$

$t = \frac{1}{2}$

$$A = \underbrace{200\left(1+\frac{0.0125}{12}\right)^{12(1/2)}}$$

$$\boxed{A \approx \$201.25}$$

Page 323 #41

George is contemplating the purchase of 100 shares of a stock selling for $15 per share. The stock pays no dividends. The history of the stock indicates that it should grow at an annual rate of 15% per year. How much will the 100 shares of stock be worth in 5 years?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = ?$

$P = 15(100) = 1500$

$r = .15$

$n = 1$

$t = 5$

$A = 1500\left(1+\frac{.15}{1}\right)^{5\cdot 1}$

$\boxed{A \approx \$3017.04}$

Page 334 #6

A culture of bacteria obeys the law of uninhibited growth. If 500 bacteria are present initially and there are 800 after 1 hour, how many will be present in the culture after 5 hours? How long is it until there are 20,000 bacteria?

#1

#2

$A = A_0 e^{kt}$

$A = 800$

$A_0 = 500$

$k = ?$

$t = 1$

① Before we can solve the first question, we must find k.

$$\frac{800}{500} = \frac{500e^{k \cdot 1}}{500}$$

$$1.6 = e^{k}$$

$$k = \ln(1.6) \approx 0.4700$$

② Now we can answer #1

$$A = 500e^{.4700 \cdot 5} \approx \boxed{5{,}243 \text{ bacteria}}$$

③ Now we can answer the second question:

$A = 20{,}000$

$A_0 = 500$

$k = 0.47$

$t = ?$

$$\frac{20{,}000}{500} = \frac{500e^{0.47t}}{500}$$

$$40 = e^{0.47t}$$

$$\ln(40) = 0.47t$$

$$t = \frac{\ln(40)}{0.47} \approx \boxed{7.8 \text{ days}}$$

Page 335 #9

The half-life of radioactive potassium is 1690 years. If 10 grams are present now, how much will be present in 50 years?

$A = A_0 e^{kt}$

① Find Growth Rate 1st

$A = 5$

$A_0 = 10$

$k = ?$

$t = 1690$

$$\frac{5}{10} = \frac{10}{10} e^{1690k}$$

$$\frac{1}{2} = e^{1690k}$$

$$\frac{\ln(1/2)}{1690} = \frac{1690k}{1690}$$

$$k \approx -0.0004$$

② Find amount after 50 years

$A = ?$

$A_0 = 10$

$k = -0.0004$

$t = 50$

$$A = 10e^{-0.0004 \cdot 50} \approx \boxed{9.8 \text{ grams}}$$

Page 336 #23

The logistic growth model

$$P(t) = \frac{\underline{1000}}{1 + 32.33e^{-\underline{0.439}t}}$$

Represents the population (in grams) of a bacterium after t hours.

a) Determine the carrying capacity $\boxed{1000\text{ g}}$

b) What is the growth rate of the bacteria? $\boxed{43.9\%}$

c) Determine the initial population size.

Plug in $t=0$

$$P(0) = \frac{1000}{1+32.33e^{-0.439(0)}} = \boxed{30\text{ g}}$$

$$P(t) = \frac{1000}{1 + 32.33e^{-0.439t}}$$

d) What is the population after 9 hours?

$$P(9) = \frac{1000}{1+32.33e^{-0.439 \cdot 9}} \approx \boxed{616.69}$$

e) How long does it take the population to reach one-half of the carrying capacity?

$$(1+32.33e^{-0.439t}) \cdot 500 = \frac{1000}{1+32.33e^{-0.439t}} \cdot (1+32.33e^{-0.439t})$$

$$\frac{500}{500}(1+32.33e^{-0.439t}) = \frac{1000}{500}$$

$$1+32.33e^{-0.439t} = 2$$

$$-1 \qquad\qquad -1$$

$$\frac{32.33e^{-0.439t}}{32.33} = \frac{1}{32.33}$$

$$e^{-0.439t} = \frac{1}{32.33}$$

$$t = \frac{\ln\left(\frac{1}{32.33}\right)}{-0.439}$$

$$\boxed{t = 7.9 \text{ hours}}$$