Lesson 5.5: Solving Exponents and Logs

The methods for solving are no different than in previous lessons, we just now must identify the best method to solve each of the following equations. Make sure to check for extraneous solutions when the variable is the base or the argument of the log.

$$\underbrace{\log_7 343}_{3} - 2\log_7 x = 5$$

$$3 - 2\log_7 x = 5$$

$$-3 \qquad\qquad -3$$

$$\frac{-2\log_7 x}{-2} = \frac{2}{-2}$$

$$7^{\log_7 x} = 7^{-1}$$

$$x = 7^{-1} = \boxed{\frac{1}{7}} \checkmark$$

$$9^{\log_9(x^2-5x-7)} = 9^{\log_9 x}$$

$$x^2 - 5x - 7 = x$$

$$-x \qquad -x$$

$$x^2 - 6x - 7 = 0$$

$$(x-7)(x+1) = 0$$

$$\boxed{x = 7} \qquad x = -1$$

extraneous

$$\log_3(x+3) + \log_3(x+2) = \log_3(5x+7)$$

condense

$$\log_3(x^2+5x+6) = \log_3(5x+7)$$

3 3

$$x^2+5x+6 = 5x+7$$

$$-5x \quad -7 \quad -5x \quad -7$$

$$x^2-1=0$$

$$(x-1)(x+1)=0$$

$$\boxed{x=1} \checkmark \qquad \boxed{x=-1} \checkmark$$

$$e^x \cdot \frac{1}{e^3} = (e^x)^5$$

$$e^x \cdot e^{-3} = e^{5x}$$

$$e^{x-3} = e^{5x}$$

$$x - 3 = 5x$$

$$4x = -3$$

$$\boxed{x = -\frac{3}{4}}$$

$$2^{2x-3} + 11 = 23$$

$$-11 \quad -11$$

$$\log_2(2^{2x-3}) = \log_2(12)$$

$$2x - 3 = \log_2(12)$$

$$+3 \qquad +3$$

$$\frac{2x}{2} = \frac{\log_2(12) + 3}{2}$$

$$\boxed{x = \frac{\log_2(12) + 3}{2}}$$

$$2^{x^2} \cdot 4^x = 8$$

Make them all the same base.

$$2^{x^2}(2^2)^x = 2^3$$

$$2^{x^2} \cdot 2^{2x} = 2^3$$

$$2^{x^2+2x} = 2^3$$

$$x^2+2x = 3$$

$$x^2+2x-3=0$$

$$(x+3)(x-1)=0$$

$$\boxed{x=-3} \checkmark \quad \boxed{x=1} \checkmark$$

$$\ln e^{x-5} = 11$$

$$x-5=11$$

$$+5 \quad +5$$

$$\boxed{x=16} \checkmark$$

$$2\log_2 x - \log_2(x+1) = 3$$

$$\underbrace{\log_2 x^2 - \log_2}_{\text{Condense}}(x+1) = 3$$

$$2\log_2\left(\frac{x^2}{x+1}\right) = 3$$

$$(x+1)\frac{x^2}{x+1} = 8(x+1)$$

$$x^2 = 8x+8$$

$$x^2 - 8x - 8 = 0$$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(-8)}}{2(1)} = \frac{8 \pm \sqrt{96}}{2} = \frac{8 \pm 4\sqrt{6}}{2} = \boxed{4 \pm 2\sqrt{6}}$$