Lesson 4.3: Finding Asymptotes and Holes

Finding Vertical Asymptotes and Holes

1. Factor the numerator and denominator
2. Identify and eliminate common factors
   -Find the x- and y-coordinate of the hole.
3. Find vertical asymptotes (After simplifying)
   -Set remaining factors in the denominator equal to 0.

Finding Horizontal and Oblique Asymptotes

$n=0 \quad d=1$

$n < d$	Horizontal: $y = 0$
$n = d$	Horizontal: $y = \frac{a}{b}$
$n > d$	Oblique: Use synthetic or long division

n = degree of the numerator
d = degree of the denominator
a = leading coefficient of numerator
b = leading coefficient of denominator

Find all holes and asymptotes.

$$f(x) = \boxed{\frac{2x+4}{x^2-4}} = \frac{2\boxed{(x+2)}}{(x-2)\boxed{(x+2)}} = \underbrace{\frac{2}{x-2}}$$

VA: $x = 2$

Hole: $x = -2$

$y = \frac{2}{-2-2} = \frac{2}{-4} = -\frac{1}{2}$

$(-2, -\frac{1}{2})$

Hole: $(-2, -\frac{1}{2})$

HA: $y = 0$

VA: $x = 2$

$$f(x) = \frac{1x^2 - 1}{2x^2 + 7x + 6} = \frac{(x-1)(x+1)}{(2x+3)(x+2)}$$

VA: $x = -3/2$, -2

$n = 2$ $a = 1$

$d = 2$ $b = 2$

Holes: None

HA: $y = 1/2$

VA: $x = -3/2, -2$

$$f(x) = \frac{2x^2 + 3}{x - 2}$$

2	2	0	3
+	↓	4	8
	2	4	11

$$2x + 4 + \frac{11}{x-2}$$

Holes: N/A

VA: $x = 2$

OA: $y = 2x + 4$

$$f(x) = \frac{5x^3}{2x^3 - 16} = \frac{5x^3}{2(x^3-8)} = \frac{5x^3}{2(x-2)(x^2+2x+4)}$$

$x = 2$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(4)}}{2(1)}$$

$$\cancel{x = \frac{-2 \pm \sqrt{-12}}{2}}$$

imaginary

Holes: None

HA: $y = \frac{5}{2}$

VA: $x = 2$

$$f(x) = \frac{2x^2 + 6x}{2x^3 - x^2 - 18x + 9} = \frac{2x\cancel{(x+3)}}{(2x-1)(x-3)\cancel{(x+3)}} = \frac{2x}{(2x-1)(x-3)}$$

$x^2(2x-1) - 9(2x-1)$

$(2x-1)(x^2-9)$

Holes: $\left(-3, -\frac{1}{7}\right)$

$\frac{2(-3)}{(2\cdot(-3)-1)(-3-3)} = \frac{-6}{42}$

VA: $x = 3, \frac{1}{2}$

HA: $y = 0$