Lesson 3.4: Solving by Factoring

How To Solve by Factoring

1. **Set the equation equal to 0.**
2. **Factor as much as possible**
3. **Set each factor equal to 0 and solve for x.**

When $ax^2 + bx + c = 0$, then solve by using

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(non-factorable)

Solve each equation.

① $x^3 + 5x^2 + 7x + 2 = 0$

$\pm \frac{2, 1}{1}$

$\pm 2, \pm 1$

② -2 | 1 $\quad 5$ $\quad 7$ $\quad 2$

$\quad\quad -2 \quad -6 \quad -2$

$\quad 1 \quad 3 \quad 1 \quad 0$

$(x+2)(x^2+3x+1) = 0$

③ $x+2=0$

$x=-2$ ✓

$1x^2+3x+1=0$

$a=1 \quad b=3 \quad c=1$

$$x = \frac{-3 \pm \sqrt{3^2-4(1)(1)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9-4}}{2} = \frac{-3 \pm \sqrt{5}}{2}$$ ✓

$x = -2, \frac{-3 \pm \sqrt{5}}{2}$

Solve each equation.

① $2x^3 - 9x^2 + 13x - 6 = 0$

② 1 | 2 −9 13 −6

↓ 2 −7 6

2 −7 6 0

$(x-1)(2x^2-7x+6)$

$(x-1)(2x-3)(x-2) = 0$

③ $x-1=0$ $\quad 2x-3=0$ $\quad x-2=0$

$x=1$ $\quad x=3/2$ $\quad x=2$

Solve each equation.

$$2x^4 + 3x^3 - 2x^2 - x = 2$$

$-2 \quad -2$

① $2x^4 + 3x^3 - 2x^2 - x - 2 = 0$

② synthetic division

	2	3	−2	−1	−2
	↓	2	5	3	2
−2	2	5	3	2	0
	↓	−4	−2	−2	
	2	1	1	0	

③ $(x-1)(x+2)(2x^2+1x+1) = 0$

$x-1$: $x = 1$

$x+2$: $x = -2$

$2x^2+x+1$: $a = 2 \quad b = 1 \quad c = 1$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(1)}}{2(2)}$$

$$= \frac{-1 \pm \sqrt{-7}}{4} = \boxed{\frac{-1 \pm i\sqrt{7}}{4}}$$

Solve each equation.

$$3x^3 - x^2 + 12x - 4 = 0$$

$x^2(3x-1) + 4(3x-1)$

$(x^2+4)(3x-1)$

$x^2+4=0$

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm 2i$

$3x-1=0$

$x = 1/3$

$(x+2i)(x-2i)(3x-1)$

Now that we can solve, we can factor down to linear factors. Factors will be in the form x subtract the zero.

$$x^3 + 2x^2 - 9x + 2$$

2	1	2	−9	2
	↓	2	8	−2
	1	4	−1	0

$(x-2)(x^2+4x-1)$

$x-2$ ✓

$x = 2$

$$x = \frac{-4 \pm \sqrt{4^2-4(1)(-1)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{20}}{2}$$

$4 \cdot 5$, $\sqrt{20} = 2\sqrt{5}$

$$= \frac{-4 \pm 2\sqrt{5}}{2}$$

$$x = -2 \pm \sqrt{5}$$

$(x-2)(x-(-2+\sqrt{5}))(x-(-2-\sqrt{5}))$

$(x-2)(x+2-\sqrt{5})(x+2+\sqrt{5})$

Now that we can solve, we can factor down to linear factors. Factors will be in the form *x* subtract the zero.

$$3x^3 + 7x^2 + 6x + 14 = 0$$

$x^2(3x+7) + 2(3x+7) = 0$

$(x^2+2)(3x+7) = 0$

Linear Factors

$(x - i\sqrt{2})(x + i\sqrt{2})(3x+7)$

Solve.

$3x+7=0$

$x = -7/3$

$x^2+2=0$

$x = \pm i\sqrt{2}$

$x^2 + 2 = 0$

$-2 \quad -2$

$\sqrt{x^2} = \sqrt{-2}$

$x = \pm i\sqrt{2}$

Directions for Homework 3.4:

1. Find all solutions (real and imaginary)
2. Factor down to linear factors.

How to Know When to Use What Method

1. Grouping Method: 4 terms
2. Diagram/Bottoms Up: 3 terms (highest exponent is twice as big as middle term's exponent)
3. Perfect Squares: 2 terms
4: Perfect cubes: 2 terms
5. Synthetic Division: Every other method fails