Lesson 3.2: Theorems About Roots/Zeros

Factor $6x^3 + 11x^2 + 6x + 1$

$x^2(6x+11) + 1(6x+1)$

They are not the same!

We have to have to use Synthetic division to factor this.

Remainder Theorem: when a polynomial $f(x)$ is divided by $x-a$, the remainder (r) equals $f(a)$. $f(a) = r$

Factor Theorem: If $f(a) = 0$, then $x-a$ is a factor of $f(x)$.

Why? $\frac{f(x)}{x-a} = q(x) + \frac{r}{x-a}$

$f(x) = (x-a) \cdot q(x) + r$

$f(a) = (a-a) \cdot q(a) + r$

$f(a) = 0 \cdot q(a) + r$

$f(a) = r$

Determine if the following if the following are factors of

$$5x^3 - 2x^2 - 4x + 1$$

1) $x - 1 = 0$

$x - a$

$a = 1$

$f(1) = 5(1)^3 - 2(1)^2 - 4(1) + 1$

$f(1) = 0$

Remainder

$\boxed{\text{Factor.}}$

* 2) $x + 3$

$a = -3$

$f(-3) = 5(-3)^3 - 2(-3)^2 - 4(-3) + 1$

$f(-3) = -140$

Remainder

$\boxed{\text{Not a factor}}$

Use the Remainder Theorem or Factor Theorem to determine whether *x-c* is a factor of *f*.

$$3x^6 - 10x^2 - 4x + 3\,;\ c = -3$$

$a = -3$

$$f(-3) = 3(-3)^6 - 10(-3)^2 - 4(-3) + 3$$
$$= 2{,}112$$

Remainder

$\boxed{\text{Not a factor}}$

Rational Roots Test:

A way of obtaining a list of useful first guesses when you are trying to find the zeros of a polynomial. Given a polynomial with integer coefficients, the potential zeros are found by the following method:

$$\pm\frac{factors\ of\ the\ constant}{factors\ of\ the\ leading\ coefficient}$$

Interchangeable Terms:

- Roots
- x-intercepts
- zeros

List all potential rational zeros of each polynomial.

* $2x^6 - 10x^2 - 4x + 15$

$\pm \dfrac{1, 3, 5, 15}{1, 2}$

$\pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}, \pm 5, \pm \frac{5}{2},$
$\pm 15, \pm \frac{15}{2}$

* $4x^5 - 11x^3 - 9x + 10$

$\pm \dfrac{1, 2, 5, 10}{1, 2, 4}$

$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2, \pm 5, \pm \frac{5}{2}, \pm \frac{5}{4}$
$\pm 10,$

Additional Zeros/Roots

<u>Complex Root Theorem</u>: if a+b*i* is a root, then a-b*i* is also a root.

$i = \sqrt{-1}$ (come in pairs)

* The same could be said for roots with square roots:

If $a + \sqrt{b}$ is a root, then $a - \sqrt{b}$ is also a root.

Why?

$\sqrt{x^2} = \sqrt{3}$

$x = \pm\sqrt{3}$

$x^2 = -3$

$x = \pm i\sqrt{3}$

A polynomial function with rational coefficients has the given zeros. List any additional zeros the polynomial must have.

* $2+\sqrt{7}$, -9, $3i$

$2-\sqrt{7}$, $-3i$

* 5, 0, -7, $\sqrt{3}$

$-\sqrt{3}$

* $7-i$, $5+3\sqrt{2}$, -11

$7+i$, $5-3\sqrt{2}$

Descartes' Rule of Signs

This rule tells you how many real roots to expect.

<u>Positive</u>: Count the number of sign changes of $f(x)$

<u>Negative</u>: Count the number of sign changes of $f(-x)$

State the possible number of positive and negative zeros using Descartes' Rule of Signs.

$$+x^5 - x^4 + 3x^3 + 9x^2 - x + 5$$

Positive: 4 or 2 or 0

$$(-x)^5 - (-x)^4 + 3(-x)^3 + 9(-x)^2 - (-x) + 5$$

$$-x^5 - x^4 - 3x^3 + 9x^2 + x + 5$$

Negative: 1

State the possible number of positive and negativer zeros using Descartes' Rule of Signs.

$$+2x^4 - x^3 + 4x^2 - 5x^1 + 3$$

Positive : 4 or 2 or 0.

$$2x^4 + x^3 + 4x^2 + 5x + 3$$

Negative: 0

State the possible number of positive and negativer zeros using Descartes' Rule of Signs.

$$3x^6 - 5x^5 + 4x^2 + 4x - 1$$