Lesson 3.2: Theorems About Roots/Zeros

We have to have to use Synthetic division to factor this. Remainder Theorem: when a polynomial f(x) is divided by x-a, the remainder (r) equals f(a). f(x) = f(x)

<u>Factor Theorem:</u> If f(a) = 0, then x-a is a factor of f(x).

Why?
$$\frac{f(x)}{x-a} = g(x) + \frac{f(x)}{x-a}$$

 $f(x) = (x-a) \cdot g(x) + f(x) = (a-a) \cdot g(a) + f(a) = (a-a) \cdot g(a) + f(a) = f(a$

Determine if the following if the following are factors of

$$5x^3 - 2x^2 - 4x + 1$$

1)
$$x - 1 = 0$$
 $x - a$
 $a = 1$

$$f(1) = 5(1)^{3} - 2(1)^{2} - 4(1) + 1$$

$$f(0) = 0$$
Remainder

1)
$$x-1=0$$
 $x-a$
 $a=-3$
 $f(1) = 5(1)^3 - 2(1)^2 - 4(1) + 1$
 $f(3) = 5(-3)^3 - 2(-3)^2 - 4(-3) + 1$
 $f(3) = 0$

Remainder

Not a factor

Use the Remainder Theorem or Factor Theorem to determine whether *x-c* is a factor of *f*.

$$3x^{6} - 10x^{2} - 4x + 3$$
; $c = -3$
 $f(-3) = 3(-3)^{6} - 10(-3)^{2} - 4(-3) + 3$
 $= 2,112$
Remainder
Not a factor

Rational Roots Test:

A way of obtaining a list of useful first guesses when you are trying to find the zeros of a polynomial. Given a polynomial with integer coefficients, the <u>potential</u> zeros are found by the following method:

$$\pm \frac{factors\ of\ the\ constant}{factors\ of\ the\ leading\ coefficient}$$

Interchangeable Terms:
- Roots
- X-intercepts
- zeros

List all potential rational zeros of each polynomial.

*
$$(2x^{6} - 10x^{2} - 4x + 15)$$
 $t = 1,3,5,15$
 $t = 1,2$
 $t = 1,3,5,15$
 $t = 1,2$
 $t = 1,3$
 $t = 1,3$

Additional Zeros/Roots

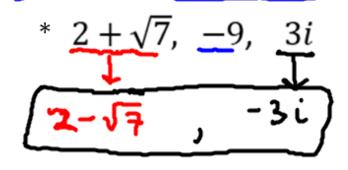
<u>Complex Root Theorem</u>: if a+bi is a root, then a-bi is also a root.

* The same could be said for roots with square roots:

If $a + \sqrt{b}$ is a root, then $a - \sqrt{b}$ is also a root.

$$Why?$$
 $X^{2}=\sqrt{3}$
 $X = \pm i\sqrt{3}$
 $X = \pm i\sqrt{3}$

A polynomial function with rational coefficients has the given zeros. List any additional zeros the polynomial must have.



*
$$\frac{7-i}{1}$$
, $\frac{5+3\sqrt{2}}{5-3\sqrt{2}}$

Descartes' Rule of Signs

This rule tells you how many real roots to expect.

<u>Positive</u>: Count the number of sign changes of f(x)

<u>Negative</u>: Count the number of sign changes of f(-x)

State the possible number of positive and negative zeros using Descartes' Rule of Signs.

$$+x^5 - x^4 + 3x^3 + 9x^2 - x + 5$$

$$(x)^{5} - (-x)^{4} + 3(-x)^{3} + 9(-x)^{2} - (-x) + 5$$

 $-x^{5} - x^{4} - 3x^{3} + 9x^{2} + x + 5$

State the possible number of positive and negativer zeros using Descartes' Rule of Signs.

$$+2x^{4}-x^{3}+4x^{2}-5x^{2}+3$$
Positive: 4 or 2 or 0.
$$2x^{4}+x^{3}+4x^{2}+5x+3$$

State the possible number of positive and negativer zeros using Descartes' Rule of Signs.

$$3x^6 - 5x^5 + 4x^2 + 4x - 1$$