

3.1: Factoring Techniques

Factoring Quadratics

There are several methods that are taught to factor quadratics with leading coefficients.

Method 1: Diagram

Use factors of the leading coefficient and the constant to find a pair that adds to the middle term.

$$6x^2 - x - 12$$

The diagram illustrates the factoring of the quadratic expression $6x^2 - x - 12$ using a box method. A blue-outlined rectangle encloses the factors $(2x - 3)$ and $(3x + 4)$. Below this rectangle, a red-outlined oval contains the middle term $-9x$. Red curved arrows point from the terms $2x$ and 3 in the factors up to the $-9x$ term. Below the oval, a red horizontal line with a plus sign above it and a minus sign below it is labeled $+8x$, with a red checkmark at the end. This indicates that the terms $2x$ and $3x$ were combined to produce the middle term $-9x$.

Factoring Quadratics

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Method 1: Diagram

$$3x^2 + 7x - 6$$
$$(3x - 2)(x + 3)$$
$$\frac{9x}{7x}$$

Factoring Quadratics

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Method 2: Grouping

Multiply the leading coefficient and constant. Then find factors of that number that add to the middle term.

$$\begin{array}{r} 2x^2 - 7x - 4 \\ \underline{-8} \\ \underline{\underline{-8, 1}} \\ \begin{array}{r} 2x^2 - 8x + 1x - 4 \\ \text{GCF: } 2x \qquad \text{GCF: } 1 \\ 2x(x-4) + 1(x-4) \\ \boxed{(x-4)(2x+1)} \end{array} \end{array}$$

Box Method

$$\begin{array}{|c|c|} \hline 2x^2 & -8x \\ \hline 1x & -4 \\ \hline x & -2 \\ \hline \end{array} \quad \begin{array}{l} 2x \\ 1 \\ -2 \end{array}$$

$$(2x+1)(x-4)$$

Special Quadratics

✗ $a^2 - b^2 = (a - b)(a + b)$ ✗

→ $(x + a)^2 = x^2 + 2ax + a^2$

→ $(x - a)^2 = x^2 - \underline{2ax} + a^2$

* $49x^2 - 16$

$(7x + 4)(7x - 4)$

* $4x^2 - 12x + 9$

$$\begin{array}{c} 4x^2 \quad 9 \\ \hline 2x \quad -3 \end{array}$$

$2 \cdot 3 = 6$

Quadratic Forms

~3 terms.

- The degree is twice as big as the second exponent

$$\underline{3x^8 + 1x^4 - 2}$$

$$(3x^4 - 2)(1x^4 + 1)$$

$$\frac{x^3}{-2}$$

$$3x^8 + 1x^4 - 2$$

$$\frac{\sim 6}{3 - 2}$$

$$\underline{3x^8 + 3x^4} - 2x^4 - 2$$

$$3x^4(x^4 + 1) - 2(x^4 + 1)$$

$$(x^4 + 1)(3x^4 - 2)$$

$$* \underline{5x^4 - 23x^2 + 12}$$

$$(5x^2 - 3)(x^2 - 4)$$

$$\frac{-3}{-20}$$

$$\underline{(5x^2 - 3)(x^2 - 4)}$$

$$\underline{(5x^2 - 3)(x + 2)(x - 2)}$$

Using Grouping Method

$$3x^8 + x^4 - 2$$

$\frac{-6}{3 \quad -2}$

$$\underline{3x^8 + 3x^4} - 2x^4 - 2$$
$$x^4(x^4 + 1) - 2(x^4 + 1)$$
$$(x^4 + 1)(3x^4 - 2)$$

Grouping Method (4 terms)

$$\underline{5x^3 + 4x^2} + 10x + 8$$

$$x^2(\underline{5x+4}) + 2(\underline{5x+4})$$

$$\boxed{(5x+4)(x^2+2)}$$

$$* \underline{4x^3 - 8x^2} - 9x + 18$$

$$4x^2(x-2) - 9(x-2)$$

$$(x-2)(4x^2-9)$$

$$\boxed{(x-2)(2x+3)(2x-3)}$$

Perfect Cubes

$$\begin{aligned} \cancel{\text{x}} \quad a^3 - b^3 &= (a - b)(a^2 + ab + b^2) \\ \cancel{\text{x}} \quad a^3 + b^3 &= (a + b)(a^2 - ab + b^2) \end{aligned}$$

$$x^3 - 8$$

$$\begin{aligned} a &= \sqrt[3]{x^3} = x \\ b &= \sqrt[3]{8} = 2 \end{aligned} \rightarrow (x - 2)(x^2 + 2x + 4)$$

$$* 27x^4 + 125x \quad G(F: x)$$

$$x(27x^3 + 125)$$

$$a = 3x$$

$$b = \sqrt[3]{125} = 5$$

$$x(3x + 5)(9x^2 - 15x + 25)$$

$$3x^3 - 6x^2 - 27x + 54$$

GCF: 3

$$3 \left(\underline{x^3 - 2x^2} \right) \overline{-9x + 18}$$

$$3 \left[x^2 \underline{(x-2)} - 9 \underline{(x-2)} \right]$$

$$3(x-2)(x^2-9)$$

$$3(x-2)(x-3)(x+3)$$

* $54x^4 - 250x$

Use Perfect Squares or
Perfect Cubes for 2 terms

GCF: $2x$

$$2x(27x^3 - 125)$$

$$\begin{aligned} a &= 3x \\ b &= 5 \end{aligned}$$

$$2x(3x-5)(9x^2 + 15x + 25)$$

$$3x^6 - 21x^4 + 36x^2$$

Quadratic

GCF: $3x^2$

$$3x^2(x^4 - 7x^2 + 12)$$

$$3x^2(x^2 - 3)(x^2 - 4)$$

$$\boxed{3x^2(x^2 - 3)(x - 2)(x + 2)}$$

$$-3x^5 - 7x^3 + 6x$$

Quadratic.

$$-x \left(\underline{3x^4 + 7x^2} \text{ } \cancel{- 6} \right)$$

$$-x \left(3x^2 - 2 \right) \left(x^2 + 3 \right)$$

-2
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