Lesson 2.3: Polynomial Division (Long Division and Synthetic Division)

Long Division

Using long division to divide polynomials is very similar to using long division to divide two numbers.

$5349 \div 7$

$$
\begin{array}{r}
764 \\
7\overline{)5349} \\
-49 \\
\hline
44 \\
-42 \\
\hline
29 \\
-28 \\
\hline
1
\end{array}
$$

$$\boxed{764\ \tfrac{1}{7}}$$

$$\frac{2x^2}{2x} = x$$

$$x(2x+1)$$

$$\frac{-4x}{2x} = -2$$

* $(2x^2 - 3x + 4) \div (2x + 1)$

$$
\begin{array}{r}
x - 2 \\
2x+1\overline{)2x^2 - 3x + 4} \\
+(-2x^2 + -x) \\
\hline
-4x + 4 \\
-(-4x - 2) \\
\hline
6
\end{array}
$$

$$\boxed{x - 2 + \frac{6}{2x+1}}$$

$$(x^4 + 5x^3 - 7x^2 + 4x + 1) \div (x^2 - 2)$$

Side Work

$$\frac{x^4}{x^2} = x^2$$

$$x^2(x^2-2)$$

$$\frac{5x^3}{x^2} = 5x$$

$$5x(x^2-2)$$

$$\frac{-5x^2}{x^2} = -5$$

$$-5(x^2-2)$$

$$x^2 + 5x - 5$$

$$x^2 - 2 \overline{\big) x^4 + 5x^3 - 7x^2 + 4x + 1}$$

$$-(x^4 \qquad - 2x^2)$$

$$5x^3 - 5x^2 + 4x + 1$$

$$-(5x^3 \qquad - 10x)$$

$$-5x^2 + 14x + 1$$

$$-(-5x^2 \qquad + 10)$$

$$14x - 9$$

$$\boxed{x^2 + 5x - 5 + \frac{14x-9}{x^2-2}}$$

$$(2x^4 + 21x^3 + 38x^2 - 76x - 25) \div (2x + 9)$$

$$\frac{2x^4}{2x} = x^3$$

$$x^3(2x+9)$$

$$\frac{12x^3}{2x} = 6x^2$$

$$6x^2(2x+9)$$

$$\frac{-16x^2}{2x} = -8x$$

$$-8x(2x+9)$$

$$\frac{-4x}{2x} = -2$$

$$-2(2x+9)$$

$$\begin{array}{r}
x^3 + 6x^2 - 8x - 2 \\
2x+9 \overline{\big) 2x^4 + 21x^3 + 38x^2 - 76x - 25} \\
-(2x^4 + 9x^3) \\
\hline
12x^3 + 38x^2 - 76x - 25 \\
-(12x^3 + 54x^2) \\
\hline
-16x^2 - 76x - 25 \\
-(-16x^2 - 72x) \\
\hline
-4x - 25 \\
-(-4x - 18) \\
\hline
-7
\end{array}$$

$$\boxed{x^3 + 6x^2 - 8x - 2 + \frac{-7}{2x+9}}$$

$$(x^4 + x^3 - 5x^2 + 10) \div (x^2 + x)$$

Synthetic Division

*Note: Long Division can be used in all polynomial division.

Synthetic division can only be used when dividing by x+c or x-c (where c represents a constant).

zero: -4

* $(x^3 + 5x^2 + 4x + 1) \div (x + 4)$

	x^3	x^2	x	c
-4	1	5	4	1
+	↓	-4	-4	0
	1	1	0	1
	x^2	x	c	R

$$1x^2 + 1x + \frac{1}{x+4}$$

$(5x^4 - 2x^2 - 1) \div (x + 1)$

	x^4	x^3	x^2	x	c
-1	5	0	-2	0	-1
+	↓	-5	5	-3	3
	5	-5	3	-3	2
	x^3				

$$\boxed{5x^3 - 5x^2 + 3x - 3 + \frac{2}{x+1}}$$

$(x^3 - 5x + 7) \div (x - 2)$

$$
\begin{array}{r|rrrr}
2 & 1 & 0 & -5 & 7 \\
+ & & 2 & 4 & -2 \\
\hline
 & 1 & 2 & -1 & 5
\end{array}
$$

$$x^2 + 2x - 1 + \frac{5}{x-2}$$