

Lesson 2.2: Sketching Polynomials

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Information Needed to Sketch Polynomials

- 1) End behavior
- 2) The y-intercept
- 3) The x-intercept(s) and the behavior of the graph at each x-intercept.

Finding x- and y-intercepts

y-int:

$$5 + (0, 5)$$

The input is always 0.
(Replace x's with 0)

$$y = 3x^2 - 4x + 5$$

Plug 0 in for x:

$$y = 3(0)^2 - 4(0) + 5$$

$$y = 5$$

x-int:

Replace y with 0.

$$0 = 3x(x-2)^2(x+1)$$

$$0 = 3x(x-2)^2(x+1)$$

$x = 0$ $x-2 = 0 \Rightarrow x = 2$ $x+1 = 0 \Rightarrow x = -1$

Example 1: Sketch $y = -2(x + 3)^2(x - 1)$

1) End Behavior:

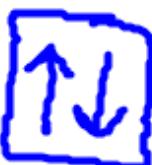
Leading Coefficient: In factored form, multiply each coefficient in front of each x.

$$\text{L.C.: } -2(1)^2(1) = \boxed{-2}$$

Degree: In factored form, count the number of x's (or multiply each x together)

$$x^2 \cdot x = x^{\textcircled{3}}$$

Degree: $\boxed{3}$

End Behavior: 

Example 1: Sketch $y = -2(x + 3)^2(x - 1)$

2) y-intercept: Plug 0 in for x.

$$= -2(0+3)^2(0-1)$$

$$= -2(3)^2(-1)$$

$$= -2(9)(-1)$$

$$= \boxed{18}$$

3) x-intercepts:

Note: -Even Multiplicity: the function **bounces** at the given x-intercept

-Odd Multiplicity: the function **crosses** at the given x-intercept.

The diagram shows the equation $0 = -2(x + 3)^2(x - 1)$. Two red arrows point from the factors $x + 3 = 0$ and $x - 1 = 0$ to a table below. The table has three columns: 'x-int.', 'Multiplicity', and 'Behavior'. The first row corresponds to $x + 3 = 0$ (so $x = -3$) with multiplicity 2 and behavior 'Bounce'. The second row corresponds to $x - 1 = 0$ (so $x = 1$) with multiplicity 1 and behavior 'Cross'.

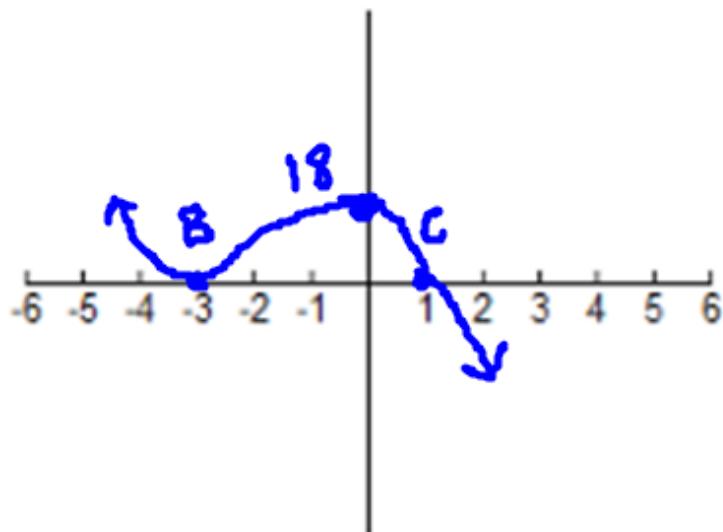
x-int.	Multiplicity	Behavior
$x + 3 = 0$	2	Bounce
$x - 1 = 0$	1	Cross

Example 1: Sketch $y = -2(x + 3)^2(x - 1)$

EB: ↑↓

y-int: 18

X-ints: -3
B
1
C



- Make sure to label the y-int.
- Make the graph smooth (no sharp points).

* C = Cross
B = Bounce

Example 2: Sketch $y = 4x^2(3 - x)(x + 4)^3$

① End Behavior: $\downarrow \downarrow$

$$\text{LC: } 4(-1)(1) = -4$$

$$\text{D: } x^2 \cdot x \cdot x^3 = x^6$$

② y-int: $y = 4(0)^2(3-0)(0+4)^3$
 $y = 0$

③ x-int: $0 = 4x^2(3-x)(x+4)^3$

M = Multiplicity

B = Bounce

C = Cross

$$x = 0$$

M: 2
B

$$3 - x = 0$$

M: 1
C

$$x + 4 = 0$$

M: 3
C

Example 2: Sketch $y = 4x^2(3 - x)(x + 4)^3$

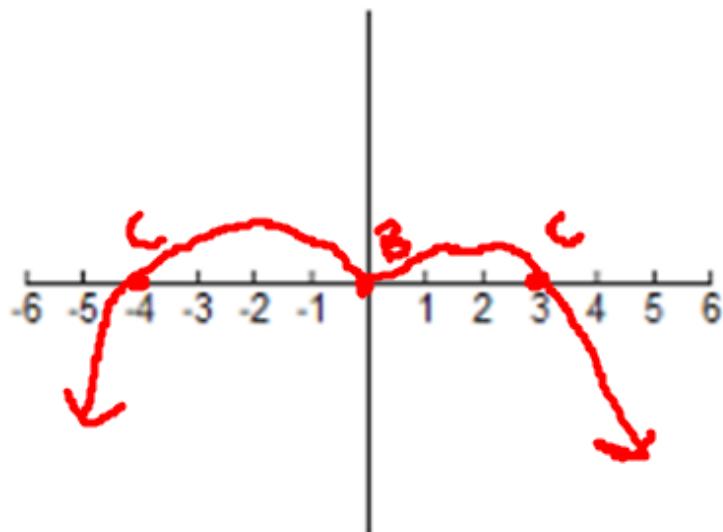
$\downarrow \downarrow$

$(0, 0)$

$(\frac{0}{8}, 0)$

$(\frac{3}{c}, 0)$

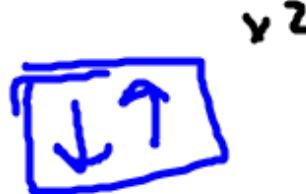
$(-\frac{4}{c}, 0)$



Example 3: Sketch $y = (x + 1)^2(x - 3)(x^2 + 3)$

① LC: 1

D: 5



② $(x + 1)^2(x - 3)(x^2 + 3)$

$$(1)^2(-3)(3) = \boxed{-9}$$

③ $\underline{(x+1)^2} \quad \underline{(x-3)} \quad \underline{(x^2+3)}$

$$x+1=0$$

$$\begin{array}{|c|} \hline x = -1 \\ M: 2 \\ B \\ \hline \end{array}$$

$$x-3=0$$

$$\begin{array}{|c|} \hline x = 3 \\ M: 1 \\ C \\ \hline \end{array}$$

$$\cancel{x^2 + 3 = 0}$$

$$\cancel{\sqrt{x^2} = \sqrt{-3}}$$

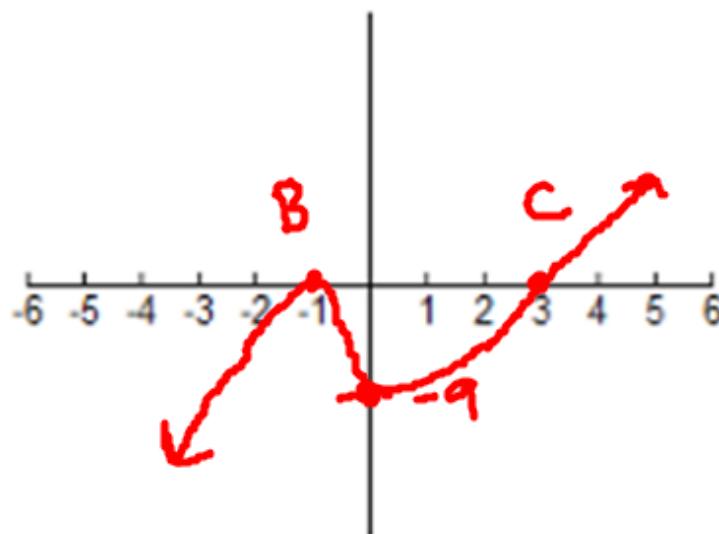
$$\cancel{x = \pm\sqrt{-3}}$$

imaginary

Example 3: Sketch $y = (x + 1)^2(x - 3)(x^2 + 3)$

$$\begin{matrix} \downarrow & \uparrow \\ y = -9 \end{matrix}$$

$$\begin{matrix} x\text{-int:} & -1 & 3 \\ & B & C \end{matrix}$$



Example 4: Sketch $f(x) = 9x - x^3$

$$= -x^3 + 9x$$

$$= -x(x^2 - 9)$$

$$= -x(x+3)(x-3)$$

↑↓

y-int: 0

x-ints: 0 -3 3

