Lesson 2.2: Sketching Polynomials

Information Needed to Sketch Polynomials

1) End behavior

2) The y-intercept

3) The *x*-intercept(s) and the behavior of the graph at each *x*-intercept.

Finding *x*- and *y*-intercepts

y-int:

The input is always 0.
(Replace x's with 0)

$$y = 3x^2 - 4x + 5$$

Plug 0 in for x:

$$y = 3(0)^2 - 4(0) + 5$$

$$y = 5$$

x-int:

Replace y with 0.

$$y = 3x(x-2)^2(x+1)$$

$$0 = \underbrace{3x}_{\boxed{x=0}}\,\underbrace{(x-2)^2}_{x-2=0,\ \boxed{x=2}}\,\underbrace{(x+1)}_{\boxed{x=-1}}$$

Example 1: Sketch $y = -2(x+3)^2(x-1)$

1) End Behavior:

<u>Leading Coefficient:</u> In factored form, multiply each coefficient in front of each x.

L.C: $-2(1)^2(1) = \boxed{-2}$

<u>Degree:</u> In factored form, count the number of x's (or multiply each x together)

$x^2 \cdot x = x^3$

Degree: $\boxed{3}$

End Behavior: $\boxed{\uparrow\downarrow}$

Example 1: Sketch $y = -2(x+3)^2(x-1)$

2) y-intercept: Plug 0 in for x.

$$= -2(0+3)^2(0-1)$$
$$= -2(3)^2(-1)$$
$$= -2(9)(-1)$$
$$= \boxed{18}$$

3) x-intercepts:

Note: -<u>Even Multiplicity</u>: the function **bounces** at the given x-intercept
-<u>Odd Multiplicity</u>: the function **crosses** at the given x-intercept.

$$0 = -2(\underbrace{x+3}_{x+3=0})^2(\underbrace{x-1}_{x-1=0})^1$$

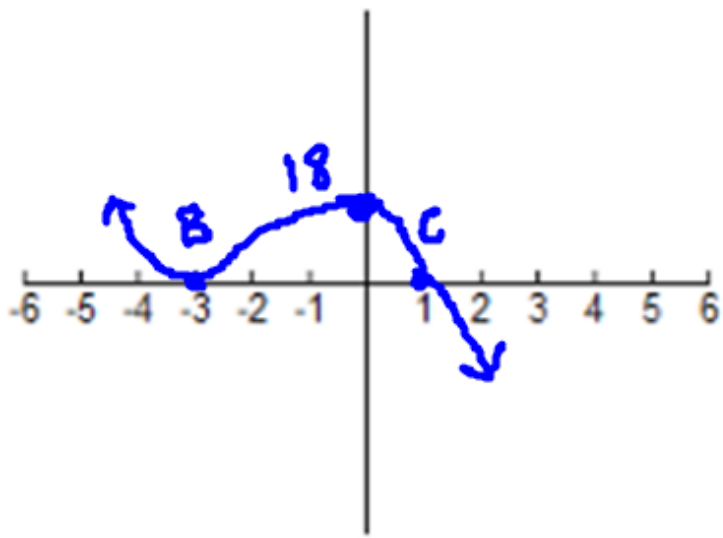
x-int.	$x=-3$	$x=1$
Multiplicity	2	1
Behavior	Bounce	Cross

Example 1: Sketch $y = -2(x+3)^2(x-1)$

EB: ↑↓

y-int: 18

x-ints: -3 B, 1 C



- Make sure to label the y-int.
- Make the graph smooth (no sharp points).

★ C = Cross
B = Bounce

Example 2: Sketch $y = 4x^2(3-x)(x+4)^3$

① End Behavior: $\boxed{\downarrow\downarrow}$

LC: $4(-1)(1) = \boxed{-4}$

D: $x^2 \cdot x \cdot x^3 = x^6$

② y-int: $y = 4(0)^2(3-0)(0+4)^3$

$\boxed{y=0}$

③ x-int: $0 = 4x^2(3-x)(x+4)^3$

$\boxed{x=0}$ M: 2 B

$3-x=0$ $\boxed{x=3}$ M: 1 C

$x+4=0$ $\boxed{x=-4}$ M: 3 C

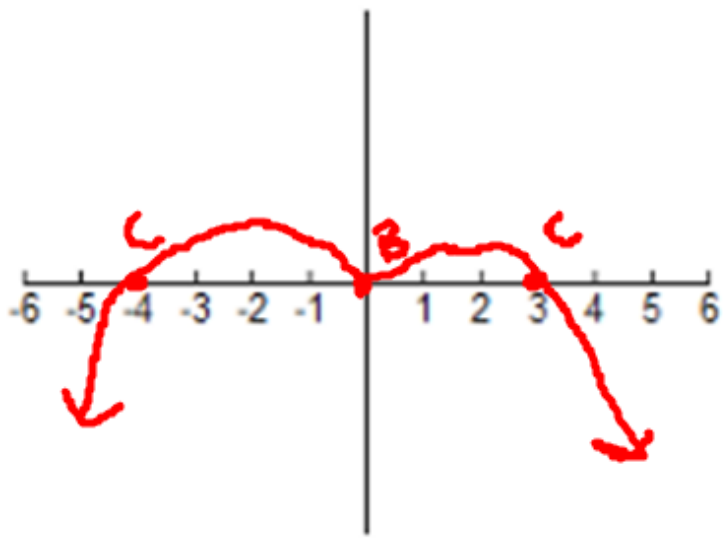
M = Multiplicity

B = Bounce

C = Cross

Example 2: Sketch $y = 4x^2(3-x)(x+4)^3$

$\downarrow\downarrow$ $(0,0)$ $(0,0)$ B $(3,0)$ C $(-4,0)$ C



Example 3: Sketch $y = (x+1)^2(x-3)(x^2+3)$

① LC: 1

D: 5

$\boxed{\downarrow\uparrow}$

② $(0+1)^2(0-3)(0^2+3)$

$(1)^2(-3)(3) = \boxed{-9}$

③ $(x+1)^2(x-3)(x^2+3)$

$x+1=0$

$\boxed{x=-1 \quad M:2 \quad B}$

$x-3=0$

$\boxed{x=3 \quad M:1 \quad C}$

$x^2+3=0$

$\sqrt{x^2}=\sqrt{-3}$

$x=\pm\sqrt{-3}$

imaginary

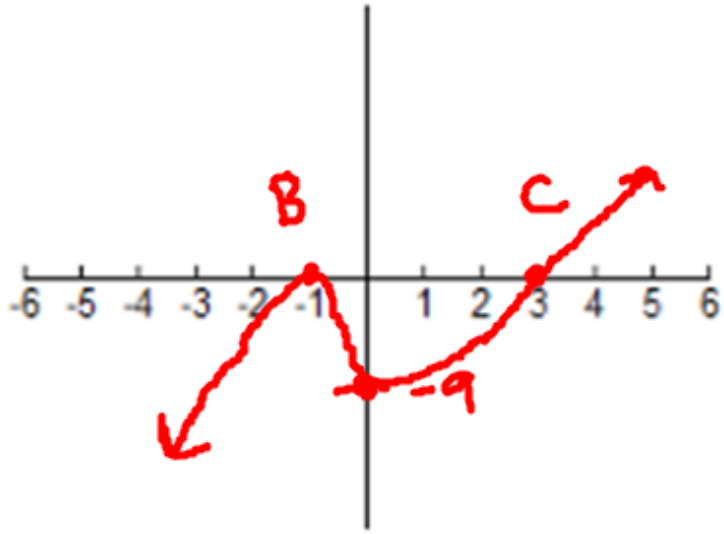
Example 3: Sketch $y = (x+1)^2(x-3)(x^2+3)$

$\downarrow\uparrow$

$y = -9$

x-int: -1 3

B C



Example 4: Sketch $f(x) = 9x - x^3$

$= -x^3 + 9x$

$= -x(x^2 - 9)$

$= -x(x+3)(x-3)$

↑↓

y-int: 0

x-int: 0 -3 3

c c c