

Lesson 2.1: Polynomials

Polynomial: An expression written in the form

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

Where

- 1) The coefficients ($a_n, a_{n-1}, \dots, a_1, a_0$) are real numbers
- 2) The exponents are nonnegative integers

Standard Form: writing a polynomial so the term with the highest exponent is written first, followed by the next highest exponent and so on.

Degree: highest exponent on the variable

Leading Coefficient: coefficient of the variable with the highest power.

End Behavior: The vertical direction of both ends of the graph

$$y = x$$



L↓R↑

$$y = x^2$$



L↑R↑

*

$$y = x^3$$



L↓R↑

$$y = x^4$$



L↑R↑

$$y = x^5$$



L↓R↑

	Even (Degree)	Odd (Degree)
+ L.C.	↑↑	↓↑
- L.C.	↓↓	↑↓

*L.C. = Leading Coefficient

$$y = -x^2$$



L↓R↓

$$y = -x^3$$



↑ ↓

*

$$y = x^2$$

 $\rightarrow 1$

$$y = \cancel{x^3} + 3x + 1$$

 $\rightarrow 2$

- * **Maximum Number of Turning Points:** one less than the degree

M#TP

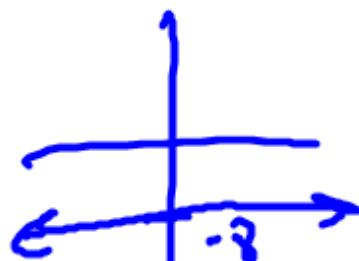
Determine if the following are polynomials. If they are, state the degree, maximum number of turning points/extrema, and the end behavior.

* $-8x^0 = -8(1) = \boxed{-8}$
Poly

Degree: 0

M#TP: 0

E.B.: $\rightarrow -8$



* $1x^2 - 7x^{99} + 4x$
Poly

D: 99
M#TP: 98

E.B. $\uparrow\downarrow$

D: odd
L.C. -7

*

	Even (Degree)	Odd (Degree)
+ L.C.	$\uparrow\uparrow$	$\downarrow\uparrow$
- L.C.	$\downarrow\downarrow$	$\uparrow\downarrow$

* L.C. = Leading Coefficient

* $\sqrt{x} + 5x + 1$

$x^{\frac{1}{2}} + 5x + 1$
Not a poly

Determine if the following are polynomials. If they are, state the degree, maximum number of turning points/extrema, and the end behavior.

	Even (Degree)	Odd (Degree)
+ L.C.	↑↑	↓↑
- L.C.	↓↓	↑↓

* L.C. = Leading Coefficient

* $\frac{4}{x} - 2$

$4x^{-1} - 2$

No
(exponent
is negative)

* $\frac{3x^2 - 5x + 12}{2}$

$\frac{3}{2}x^2 - \frac{5}{2}x + 6$

Poly

D: 2

M ≠ TP: 1

E.B: ↑↑

* $7 - \pi x + 4x^3$

Poly

E.B: ↓↑

D: odd (3)

L.C: 4 (+)

Determine if the following are polynomials. If they are, state the degree, maximum number of turning points/extrema, and the end behavior.

	Even (Degree)	Odd (Degree)
+ L.C.	↑↑	↓↑
- L.C.	↓↓	↑↓

*L.C. = Leading Coefficient

* $5x^{13} - 8x^2 + 2x^{3/2}$

No.
fraction
exponent.

Perform the given operation. Then, determine the degree, maximum number of turning points, and the end behavior.

	Even (Degree)	Odd (Degree)
+ L.C.	↑↑	↓↑
- L.C.	↓↓	↑↓

* L.C. = Leading Coefficient

*
$$(x^5 - \cancel{5x^2} + 4x + 1) + (x^4 + \cancel{5x^2} - 2x - 3)$$

$$\boxed{x^5 + x^4 + 2x - 2}$$

D: 5

M \neq TP: 4

EB: ↓↑

*
$$(x^2 - 5)(7x + 4)$$

$$\boxed{7x^3 + 4x^2 - 35x - 20}$$

D: 3

EB: ↓↑

M \neq TP: 2

Perform the given operation. Then, determine the degree, maximum number of turning points, and the end behavior.

	Even (Degree)	Odd (Degree)
+ L.C.	↑↑	↓↑
- L.C.	↓↓	↑↓

*L.C. = Leading Coefficient

* $(x^3 - 4x^2 + 1) - (5x^3 - 5x^2 - x)$

$$\begin{array}{r} \underline{x^3 - 4x^2 + 1} \\ - \underline{5x^3 + 5x^2 + x} \\ \hline - 4x^3 + 1x^2 + x + 1 \end{array}$$

D: 3

M#TP: 2

EB: ↑↓

* $(x^2 - 2x + 3)(x^3 + 3x - 4)$

$$\begin{array}{r} \underline{x^5 + 3x^3 - 4x^2 - 2x^4} \\ - \underline{6x^2 + 8x + 3x^3 + 9x - 12} \\ \hline x^5 - 2x^4 + 6x^3 - 10x^2 + 17x - 12 \end{array}$$

D: 5

M#TP: 4

EB: ↓↑