Lesson 13.1: Linear Programming

Linear Programming: a mathematical technique for maximizing or minimizing the objective function (shown below) based on constraints.

$$z = Ax + By$$

Feasible Points: The coordinates (x, y) that satisfies the system of linear inequalities or constraints.

The Solution: The feasibile point(s) that maximizes or minimizes the objective function.

Linear Programming:

1. Identify the Two Variables
2. Construct the Objective Function
   (What are you trying to maximize/minimize?)
3. Construct the Constraints (or Inequalities)
4. Identify Vertices of the Shaded Region
5. Identify the Vertex that Maximizes/Minimizes the Objective Function.

In a factory, machine 1 produces 8-inch pliers at the rate of 60 units per hour and 6-inch pliers at the rate of 70 units per hour. Machine 2 produces 8-inch pliers at a rate of 40 units per hour and 6-inch pliers at the rate of 20 units per hour. It costs $50 per hour to operate machine 1, and machine 2 costs $30 per hour to operate. The production schedule requires that at least 240 units of 8-inch pliers and at least 140 units of 6-inch pliers be produced during the 10-hour day. Which combination of machines will cost the least money to operate?

x - Machine 1 hrs

y - Machine 2 hrs

$$\text{Cost} = 50x + 30y$$

$$\begin{cases} x + y \le 10 \\ x \ge 0 \\ y \ge 0 \\ 60x + 40y \ge 240 & 8'' \\ 70x + 20y \ge 140 & 6'' \end{cases}$$

$x \ge 0$

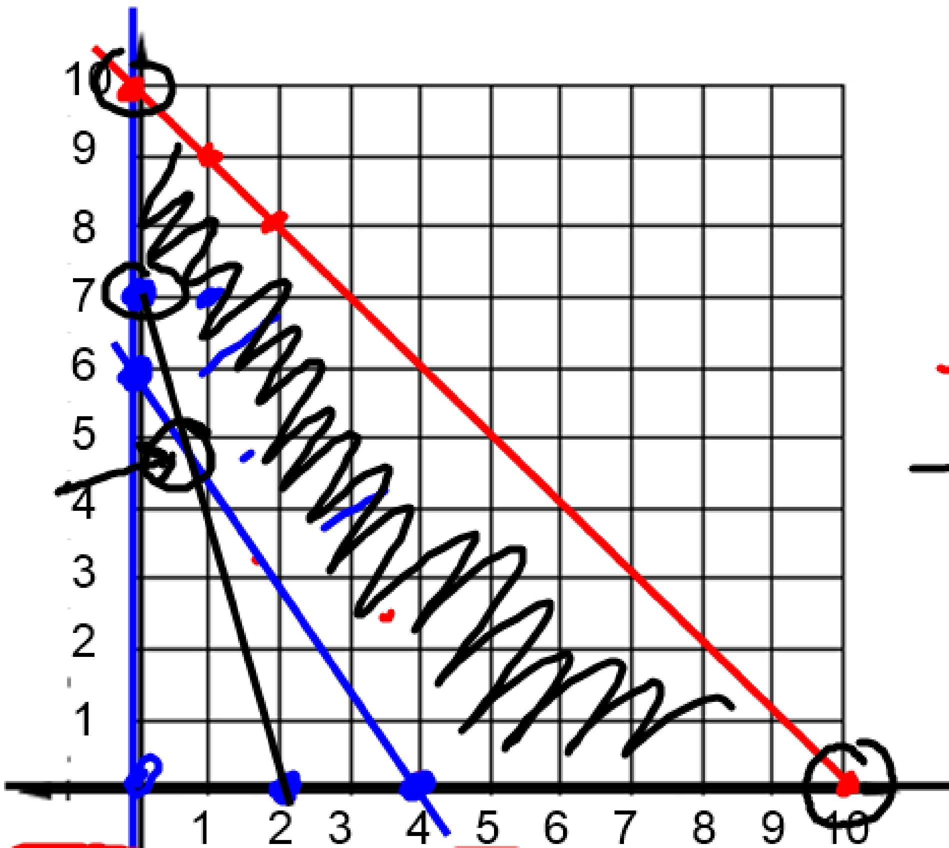
$y \ge 0$

$x + y \le 10 \rightarrow y \le -x + 10$

$60x + 40y \ge 240$

$70x + 20y \ge 140$

$\text{Cost} = 50x + 30y$



$(0.5, 5.25) \rightarrow \$100$

$(10, 0) \rightarrow \$500$

$(0, 10) \rightarrow \$300$

$(4, 0) \rightarrow \$200$

$(0, 7) \rightarrow \$210$

Machine 1

1/2 hr

Machine 2

5 hrs 15 min

$$60x + 40y = 240$$

$$(70x + 20y = 140) - 2$$

$$\begin{array}{r} 60x + 40y = 240 \\ + \; -140x - 40y = -280 \\ \hline -80x = -40 \end{array}$$

$$\frac{-80x}{-80} = \frac{-40}{-80}$$

$$x = 0.5$$

$$y = 5.25$$