Lesson 12.4: Parametric Equations

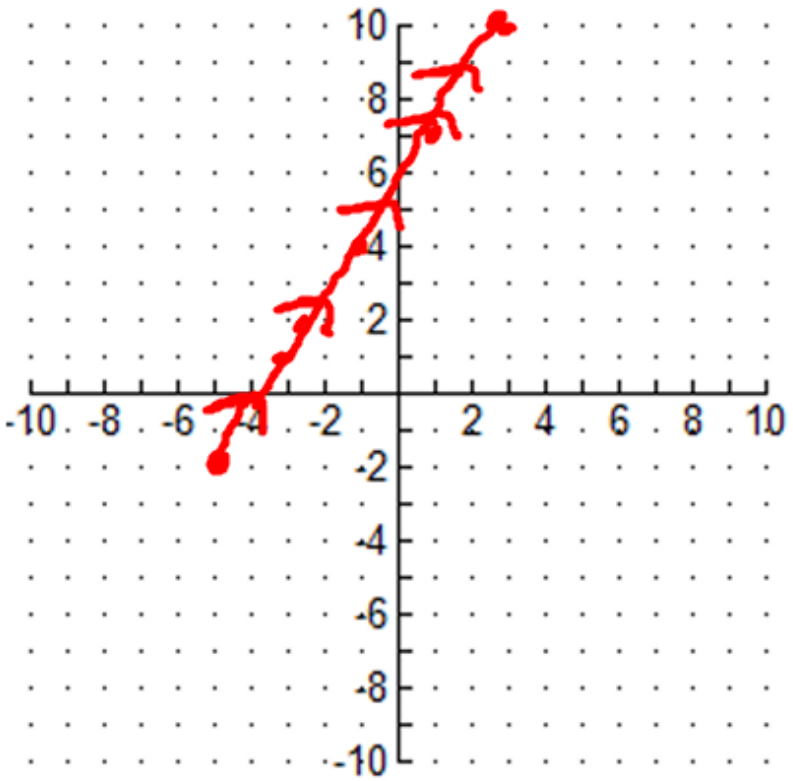
Typically, we define y in terms of x or vice versa. However, in a parametric equation, we define both x and y in terms of a third variable called a parameter. For example, assume $x = f(t)$ and $y = g(t)$. The functions $f(t)$ and $g(t)$ are parametric equations where t is the parameter.

Graph. Then find the rectangular equation.

$$x = 2t - 5$$

$$y = 3t - 2$$

$0 \le t \le 4$



Orientation (or direction of the curve)

Graph

t	x	y
0	-5	-2
1	-3	1
2	-1	4
3	1	7
4	3	10

$x = 2t - 5$ → $x + 5 = 2t$ → $t = \frac{x}{2} + \frac{5}{2}$

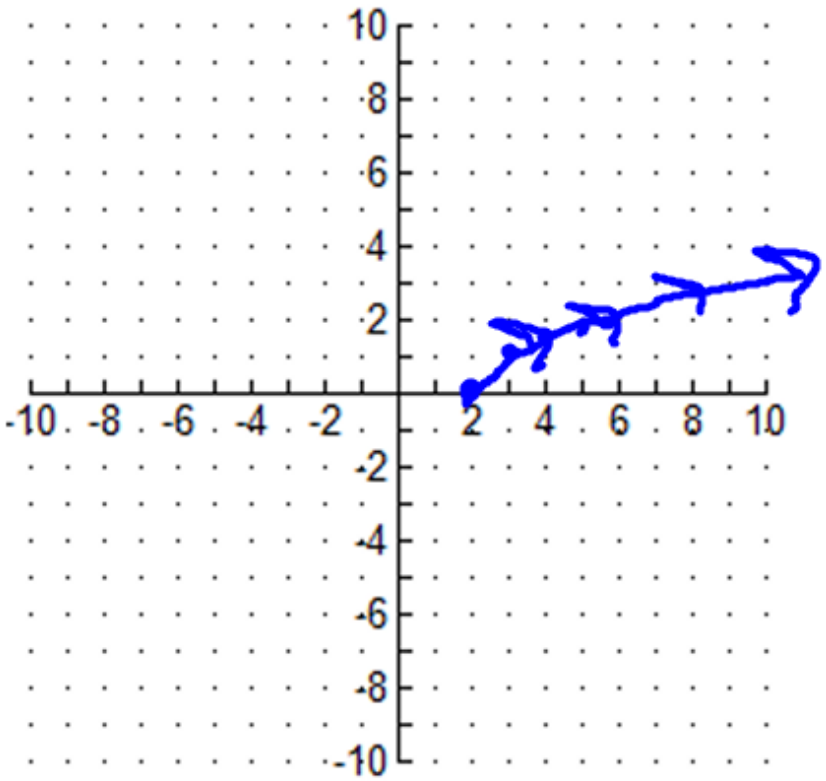
$y = 3t - 2$

$$y = 3\left(\frac{x}{2} + \frac{5}{2}\right) - 2$$

$$y = \frac{3x}{2} + \frac{15}{2} - 2$$

$$y = \frac{3x}{2} + 11/2$$

Graph. Then find the rectangular equation.



$x = t + 3$

$y = \sqrt{t + 1}$

$-1 \leq t$

t	x	y
-1	2	0
0	3	1
1	4	1.4
2	5	1.7

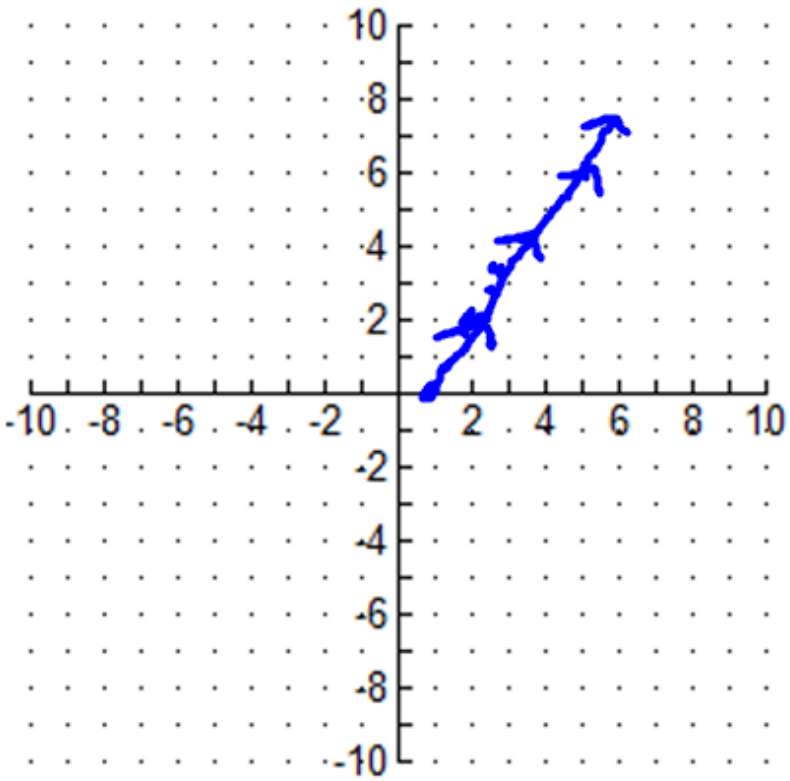
$x = t + 3$ → $t = x - 3$

$y = \sqrt{t + 1}$

$y = \sqrt{x - 3 + 1}$

$y = \sqrt{x - 2}$

Graph. Then find the rectangular equation.



$x = 1 + \sqrt{t}$ $\quad 0 \leq t$

$y = 2\sqrt{t}$

t	x	y
0	1	0
1	2	2
2	2.4	2.8
3	2.7	3.4

$x = 1 + \sqrt{t}$ → $\sqrt{t} = x - 1$

$y = 2\sqrt{t}$

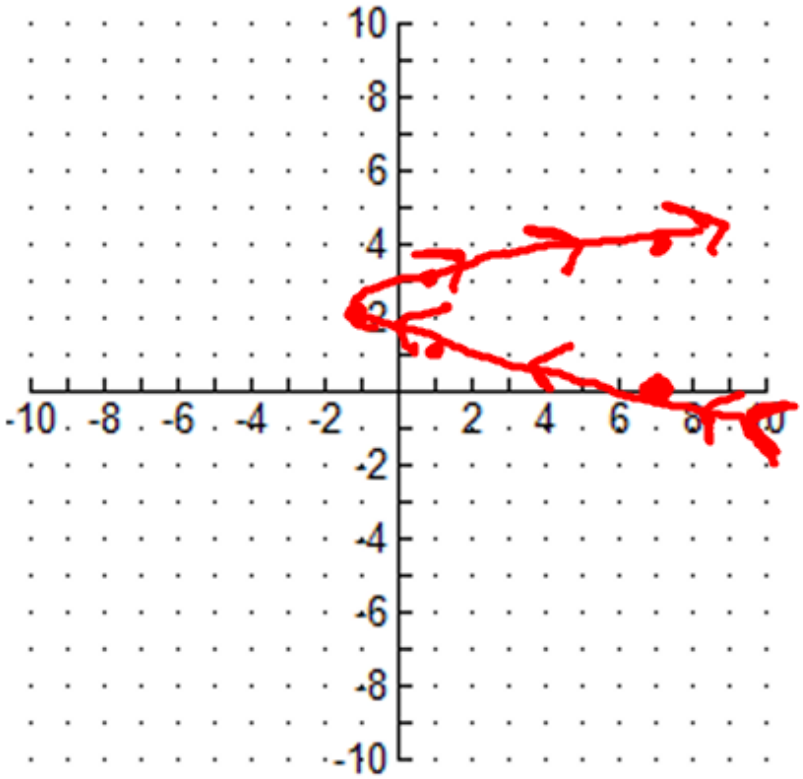
$y = 2(x-1)$

Graph. Then find the rectangular equation.

$$x = 2t^2 - 1$$

$$y = t + 2$$

$-\infty < t < \infty$



t	x	y
-2	7	0
-1	1	1
0	-1	2
1	1	3
2	7	4

$x = 2t^2 - 1$

$y = t + 2 \rightarrow t = y - 2$

$$x = 2(y-2)^2 - 1$$

Write a pair of parametric equations for each of the rectangular equations.

(Note: Be creative, do not use $x=t$ or $y=t$).

$$y = 2x + 7$$

$x = t+1$ ← made this up

$y = 2(t+1) + 7$

$y = 2t+9$

Write a pair of parametric equations for each of the rectangular equations.

(Note: Be creative, do not use $x=t$ or $y=t$).

$$y = x^2 - 5$$

$x = \sqrt{t+1}$

$y = (\sqrt{t+1})^2 - 5$

$y = t + 1 - 5$

$y = t - 4$

Farkas Bolyai's letter to his son

You must not attempt this approach to parallels: I know this way to its very end. I have traversed this bottomless night, which extinguished all light and joy of my life. For God's sake! I entreat you leave parallels alone, abhor them like indecent talk, they may deprive you [just like me] from your time, health, tranquility and the happiness of your life. That bottomless darkness may devour a thousand tall towers of Newton and it will never bright up in the earth. . . . I thought I would sacrifice myself for the sake of the truth. I was ready to become a martyr who would remove the flaw from geometry and return it purified to mankind. I accomplished monstrous, enormous labors; my creations are far better than those of others and yet I have not achieved complete satisfaction. . . . I turned back when I saw that no man can reach the bottom of this night. I turned back unconsoled, pitying myself and all mankind. . . . I have traveled past all reefs of this infernal Dead Sea and have always come back with broken mast and torn sail. The ruin of my disposition and my fall date back to this time. I thoughtlessly risked my life and happiness — *aut Caesar aut nihil.*

This is why Farkas is my favorite mathematician. Look at that passion!