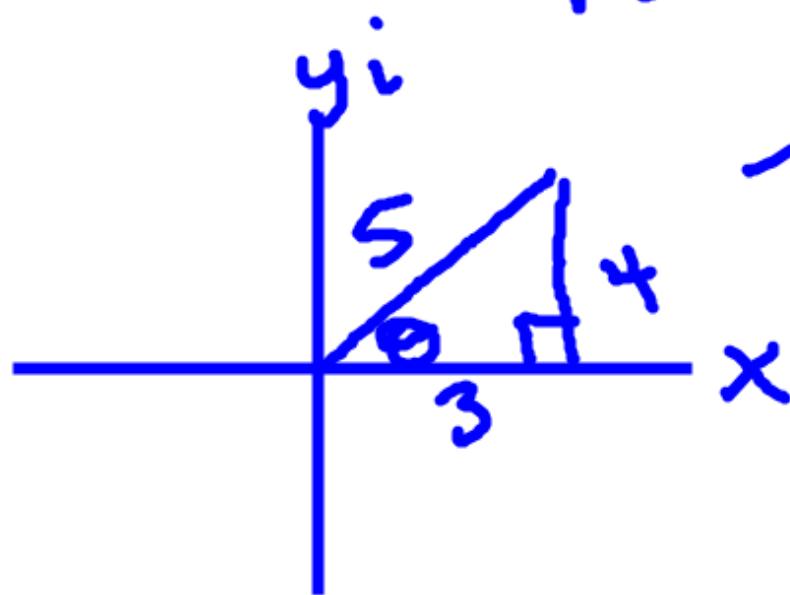


## Lesson 12.3: Complex Calculations

Complex Number  
(Rectangular Form)

47  $x + iy$



$3 + 4i$

Complex Number  
(Polar Form)

$r(\cos \theta + i \sin \theta)$

\*Shorthand:  
 $r \text{ cis } \theta$

$$z = r_1 \operatorname{cis} \theta_1$$

$$w = r_2 \operatorname{cis} \theta_2$$

\*  $zw = r_1 r_2 \operatorname{cis}(\theta_1 + \underline{\theta_2})$

$$\frac{z}{w} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

### Powers of Complex Numbers

$$[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)$$

Shorthand:  $[\operatorname{cis} \theta]^n = r^n \operatorname{cis} n\theta$

Find  $zw$  and  $\frac{z}{w}$ . Leave the answer in polar form.

$$z = 10(\cos 85^\circ + i \sin 85^\circ) = 10 \text{cis}(85^\circ)$$

$$w = 5(\cos 20^\circ + i \sin 20^\circ) = 5 \text{cis}(20^\circ)$$

$$\begin{aligned} zw &= 50 \text{cis}(85+20) \\ &= 50 \text{cis}(105^\circ) \end{aligned}$$

$$\begin{aligned} \frac{z}{w} &= \frac{10}{5} \text{cis}(85-20) \\ &= 2 \text{cis}(65^\circ) \end{aligned}$$

Find  $zw$  and  $\frac{z}{w}$ . Leave the answer in polar form.

$$z = 36 \left( \cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12} \right) = 36 \text{cis} \left( \frac{5\pi}{12} \right)$$

$$w = 9 \left( \cos \frac{\pi}{8} + i \sin \frac{\pi}{8} \right) = 9 \text{cis} \left( \frac{\pi}{8} \right)$$

$$zw = 9 \cdot 36 \text{cis} \left( \frac{5\pi}{12} + \frac{\pi}{8} \right)$$

$$324 \text{cis} \left( \frac{10\pi}{24} + \frac{3\pi}{24} \right) = \boxed{324 \text{cis} \left( \frac{13\pi}{24} \right)}$$

$$\frac{z}{w} = \frac{36}{9} \text{cis} \left( \frac{5\pi}{12} - \frac{\pi}{8} \right)$$

$$= \boxed{4 \text{cis} \left( \frac{7\pi}{24} \right)}$$

Write each expression in the standard form  $a + bi$  rectangular

$$[\sqrt{3}(\cos 15^\circ + i \sin 15^\circ)]^4$$
$$= [\sqrt{3} \operatorname{cis}(15^\circ)]^4 \quad (\sqrt{3})^2 (\sqrt{3})^2$$

$$[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta) \quad 3 \cdot 3$$

$$(\sqrt{3})^4 \operatorname{cis}(15^\circ \cdot 4)$$

$$9 \operatorname{cis}(60^\circ)$$

$$x+iy \rightarrow x = r \cos \theta$$

$$x = 9 \cos(60^\circ)$$

$$x = 9 \left(\frac{1}{2}\right)$$

$$x = 9/2$$

$$\boxed{\frac{9}{2} + i \frac{9\sqrt{3}}{2}}$$

$$y = r \sin \theta$$
$$y = 9 \sin(60^\circ)$$
$$y = 9 \left(\frac{\sqrt{3}}{2}\right)$$
$$y = 9\sqrt{3}/2$$

Write each expression in the standard form  $a + bi$ .

$$\left[2\left(\cos \frac{4\pi}{15} + i \sin \frac{4\pi}{15}\right)\right]^5 = \left[2 \operatorname{cis}\left(\frac{4\pi}{15}\right)\right]^5$$

$$= 2^5 \operatorname{cis}\left(\frac{4\pi}{15} \cdot 5\right) = 32 \operatorname{cis}\left(\frac{4\pi}{3}\right)$$

$$\begin{matrix} a+bi \\ x+yi \end{matrix}$$

$$\begin{aligned} x &= 32 \cos\left(\frac{4\pi}{3}\right) \\ &= 32\left(-\frac{1}{2}\right) = -16 \end{aligned}$$

$$\begin{aligned} y &= 32 \sin\left(\frac{4\pi}{3}\right) \\ &= 32\left(-\frac{\sqrt{3}}{2}\right) = -16\sqrt{3} \end{aligned}$$

$$\boxed{-16 - 16\sqrt{3}i}$$

Write each expression in the standard form  $a + bi$ .

$$\frac{x}{(2+2i)^3}$$

$$r^2 = 2^2 + 2^2 = 8$$

$$r = 2\sqrt{2}$$

$$x^2 + y^2 = r^2$$

$$\tan \theta = \frac{y}{x}$$

$$\tan \theta = \frac{2}{2}$$

$$\tan \theta = 1 \quad QI$$

$$\theta = \frac{\pi}{4}$$

Polar  $\left[ \underline{2\sqrt{2}} \operatorname{cis} \left( \frac{\pi}{4} \right) \right]^3$

$$(2\sqrt{2})(2\sqrt{2})(2\sqrt{2})$$

$$= (\underline{2\sqrt{2}})^3 \operatorname{cis} \left( \frac{\pi}{4} \cdot 3 \right)$$

$$8 \cdot 2\sqrt{2}$$

$$= 16\sqrt{2} \operatorname{cis} \left( \frac{3\pi}{4} \right)$$

$$16\sqrt{2} \text{ cis } \left(\frac{3\pi}{4}\right)$$

$$x = r \cos \theta$$

$$x = 16\sqrt{2} \cos\left(\frac{3\pi}{4}\right)$$

$$x = 16\sqrt{2} \left(-\frac{\sqrt{2}}{2}\right)$$

$$x = -16$$

$$y = r \sin \theta$$

$$y = 16\sqrt{2} \sin\left(\frac{3\pi}{4}\right)$$

$$y = 16\sqrt{2} \left(\frac{\sqrt{2}}{2}\right)$$

$$y = 16$$

$$\boxed{-16 + 16i}$$

## Complex Roots

$$\sqrt[n]{r \operatorname{cis} \theta} = \sqrt[n]{r} \operatorname{cis} \left( \frac{\theta}{n} + \frac{360^\circ}{n} k \right)$$

"n" number of  
answers

K = integer (0, 1, 2, ...)

Find all the complex roots. Leave your answers in polar form with the argument in degrees.

Square Roots of  $\underline{4 + 4i}$

$$r^2 = 4^2 + 4^2$$

$$r = 4\sqrt{2}$$

$$x^2 + y^2 = r^2$$

$$\tan \theta = \frac{y}{x}$$

$$\tan \theta = \frac{4}{4}$$

$$\theta = 45^\circ$$

$$((2)^{1/2})^{1/2}$$

$$4\sqrt{2} \text{ cis}(45^\circ)$$

$$\downarrow \text{square Root} \rightarrow n=2$$

$$\sqrt{4\sqrt{2}} \text{ cis}\left(\frac{45}{2} + \frac{360}{2} \cdot k\right)$$

$$2\sqrt[4]{2} \text{ cis}(22.5 + 180k)$$

$$k=0 : 2\sqrt[4]{2} \text{ cis}(22.5^\circ)$$

$$k=1 : 2\sqrt[4]{2} \text{ cis}(202.5^\circ)$$

Find all the complex roots. Leave your answers in polar form with the argument in degrees.

~~Fourth Roots of~~ Roots of ~~-81i~~

$$x = 0 \quad y = -81$$

$$r^2 = 0^2 + (-81)^2$$

$$r = 81$$

$$\sqrt[4]{81} \text{ cis } (270^\circ)$$

↓  
4th Root  $n=4$

$$\sqrt[4]{81} \text{ cis } \left( \frac{270^\circ}{4} + \frac{360^\circ}{4} k \right)$$

$$3 \text{ cis } (67.5^\circ + 90^\circ k)$$

$$k=0: 3 \text{ cis } (67.5^\circ)$$

$$k=1: 3 \text{ cis } (157.5^\circ)$$

$$x^2 + y^2 = r^2$$

$$\tan \theta = \frac{y}{x}$$

$$\tan \theta = \frac{-81}{0} = \text{undefined}$$

$$\theta = 270^\circ$$

$$k=2: 3 \text{ cis } (247.5^\circ)$$

$$k=3: 3 \text{ cis } (337.5^\circ)$$