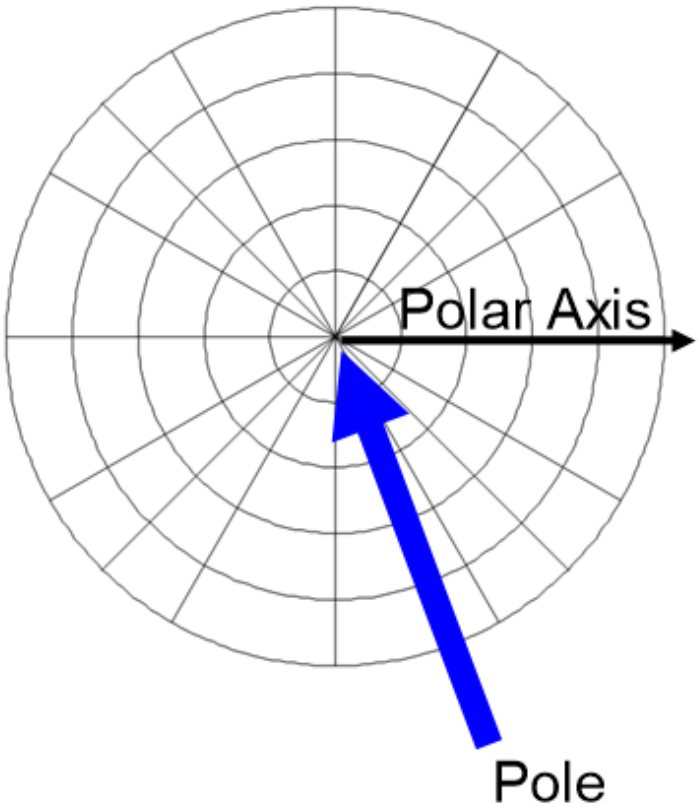
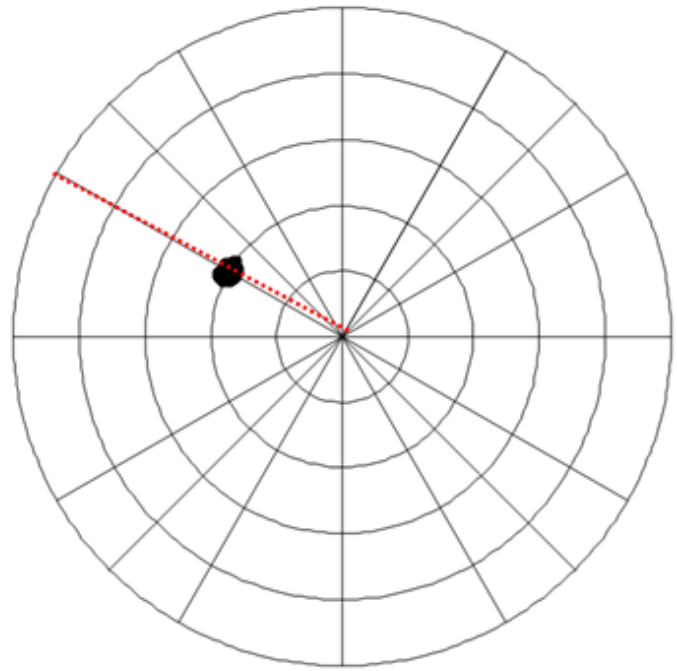
# Lesson 12.1: The Polar Coordinate System

Polar Coordinates:

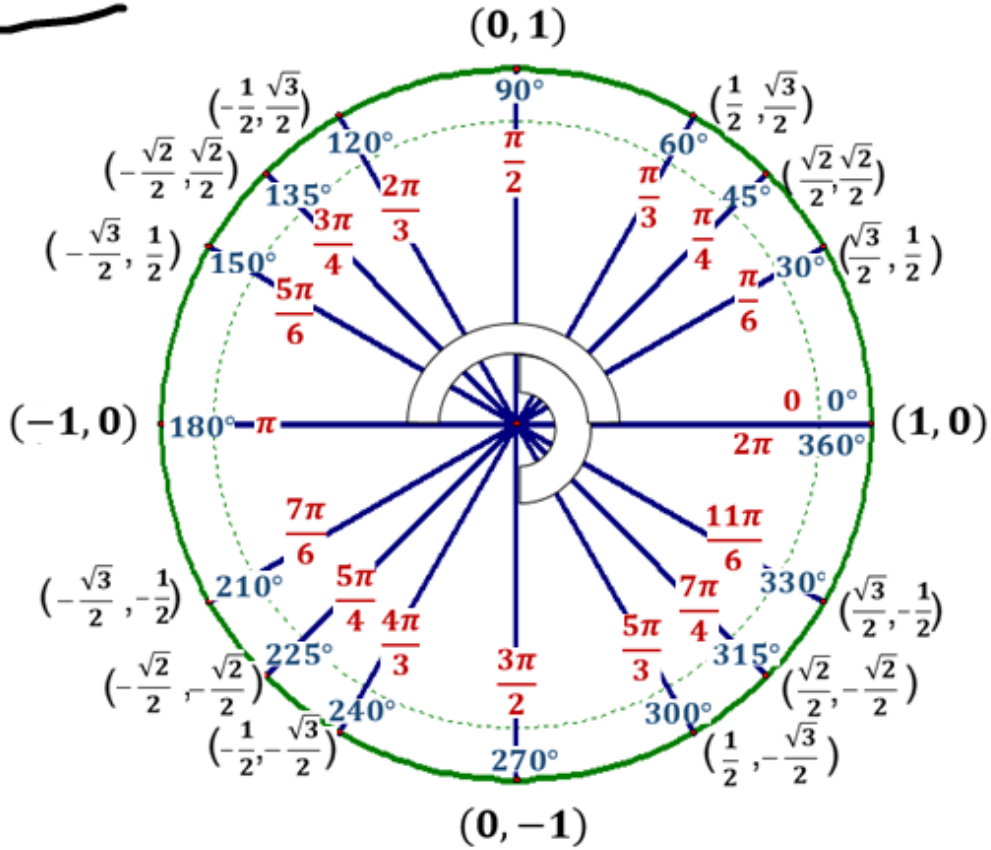
$$(r, \theta)$$



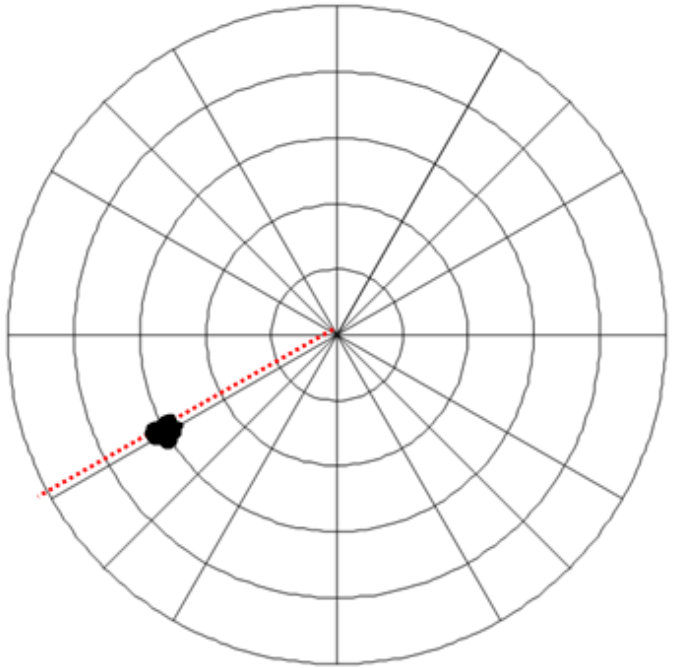



$$\left(2, \frac{3\pi}{4}\right)$$

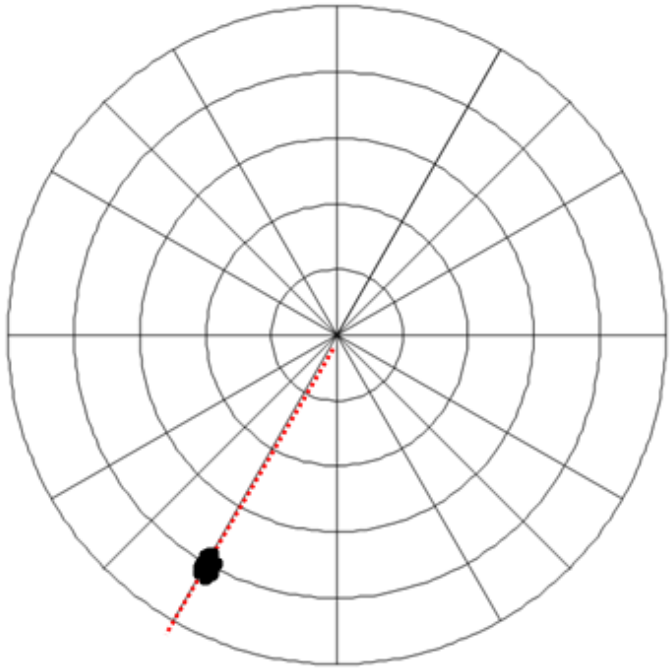
# Remember!


(0, 1)
90°
π/2
(1/2, √3/2)
60°
π/3
(√2/2, √2/2)
45°
π/4
(√3/2, 1/2)
30°
π/6
(1, 0)
0°
0
360°
2π
(−1, 0)
180°
π
(−1/2, √3/2)
120°
2π/3
(−√2/2, √2/2)
135°
3π/4
(−√3/2, 1/2)
150°
5π/6
(−√3/2, −1/2)
210°
7π/6
(−√2/2, −√2/2)
225°
5π/4
(−1/2, −√3/2)
240°
4π/3
(0, −1)
270°
3π/2
(1/2, −√3/2)
300°
5π/3
(√2/2, −√2/2)
315°
7π/4
(√3/2, −1/2)
330°
11π/6

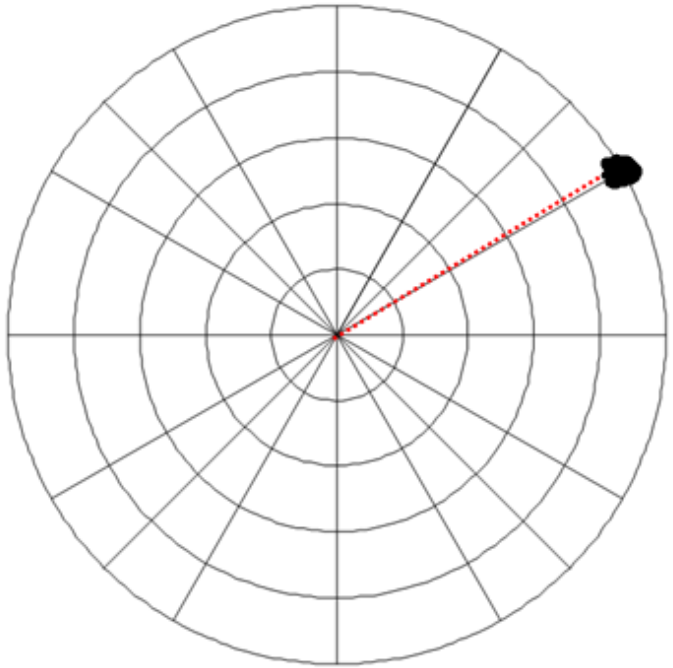
$$\left(3, \frac{7\pi}{6}\right)$$



$$\left(4, -\frac{2\pi}{3}\right)$$



$$\left(5, \frac{13\pi}{3}\right)$$

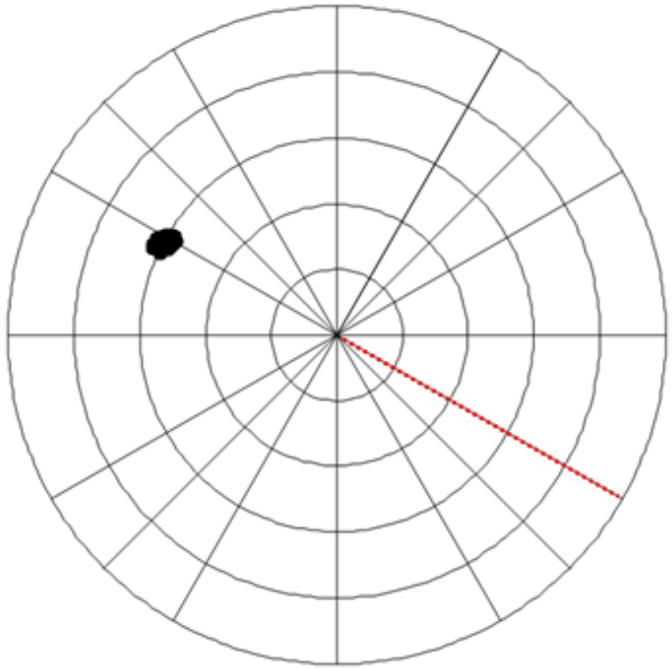


$$\frac{13\pi}{3} - 2\pi - 2\pi$$

$$\downarrow$$

$$\frac{13\pi}{3} - \frac{6\pi}{3} - \frac{6\pi}{3} = \frac{1\pi}{3}$$

$$\left(-3, \frac{11\pi}{6}\right)$$

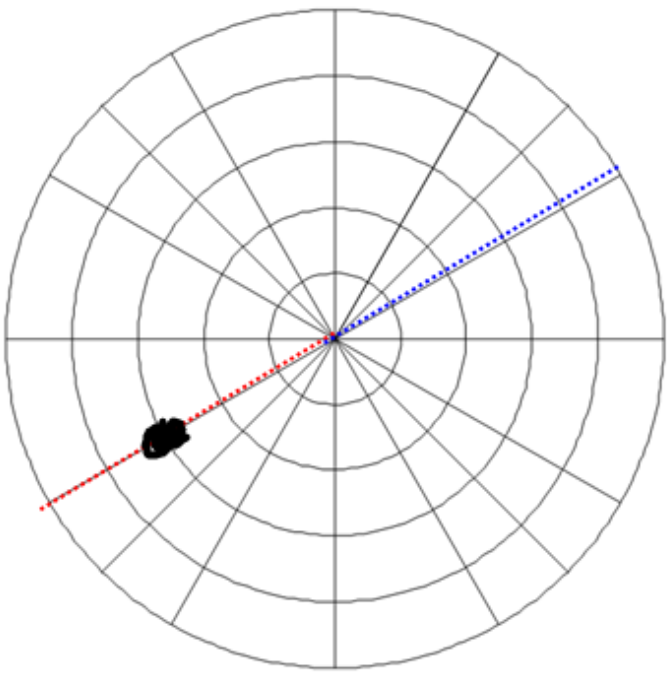


Find other polar coordinates $(r, \theta)$ of the point for which:

a) $r > 0,\ -2\pi \le \theta < 0$

b) $r < 0,\ 0 \le \theta < 2\pi$

c) $r > 0,\ 2\pi \le \theta < 4\pi$



$$\left(3, \frac{7\pi}{6}\right)$$

a) $\left(3, -\frac{5\pi}{6}\right)$

$$\frac{7\pi}{6} - \frac{12\pi}{6} = -\frac{5\pi}{6}$$

b) $\left(-3, \frac{\pi}{6}\right)$

c) $\left(3, \frac{19\pi}{6}\right)$

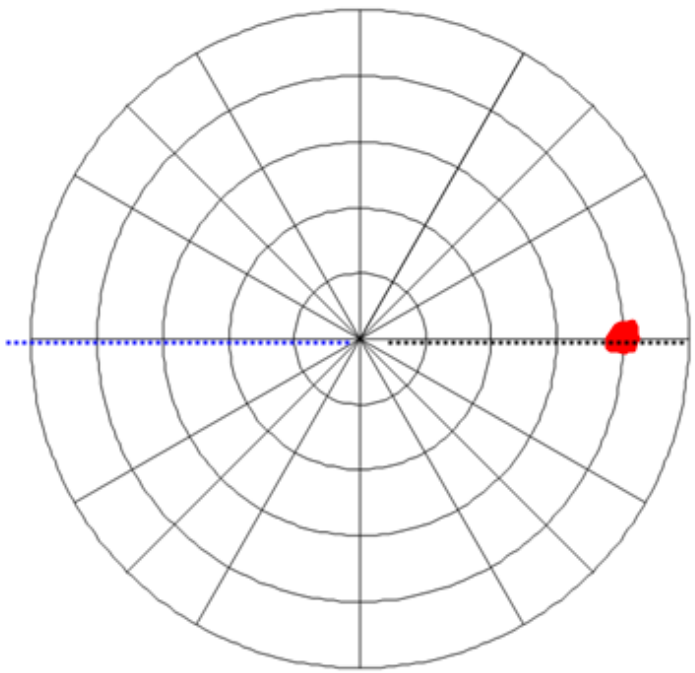
$$\frac{7\pi}{6} + \frac{12\pi}{6} = \frac{19\pi}{6}$$

Find other polar coordinates $(r, \theta)$ of the point for which:

a) $r > 0,\ -2\pi \leq \theta < 0$

b) $r < 0,\ 0 \leq \theta < 2\pi$

c) $r > 0,\ 2\pi \leq \theta < 4\pi$

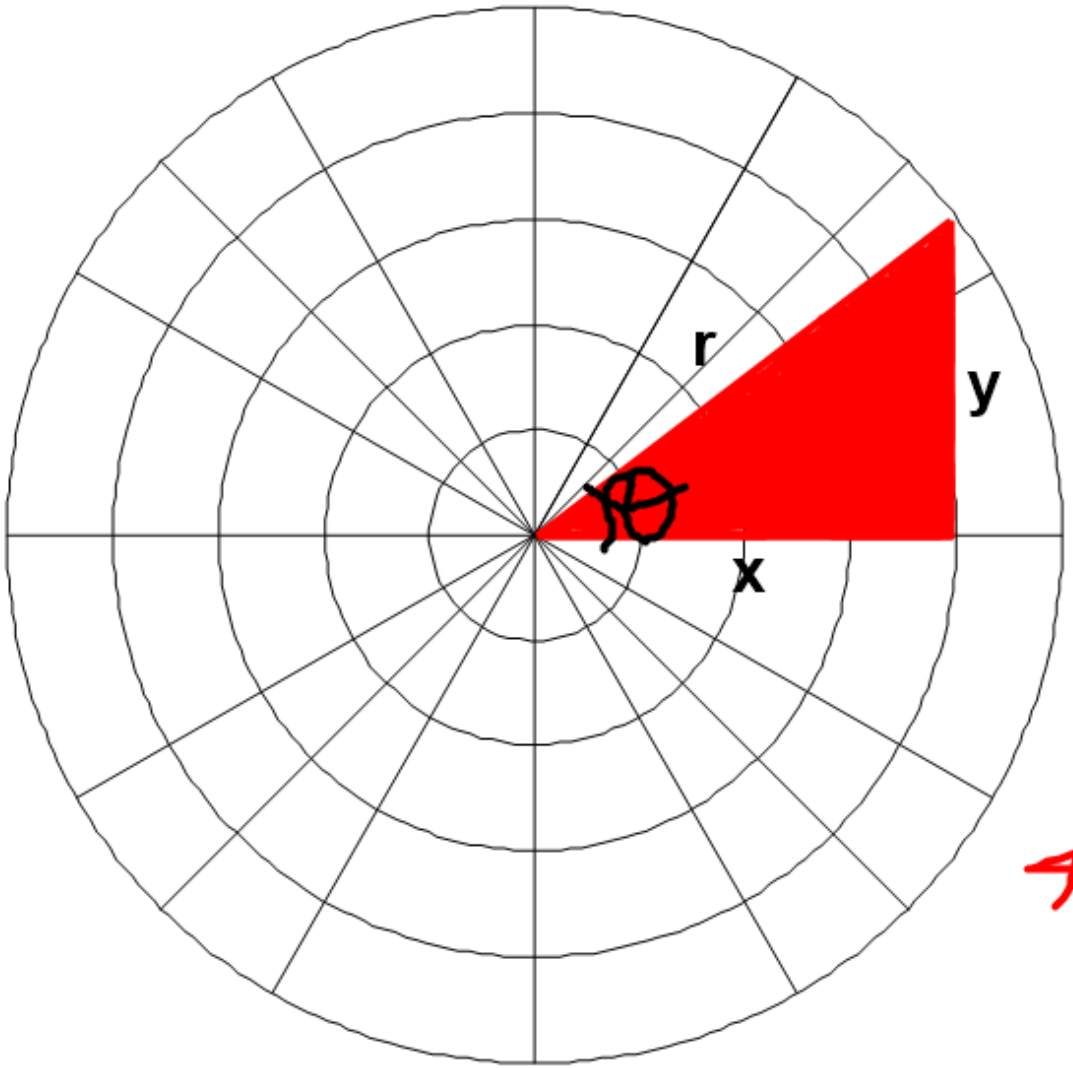


$(-4, 3\pi)$

a) $(4, -2\pi)$

b) $(-4, \pi)$

c) $(4, 2\pi)$




$$\cos\theta = \frac{x}{r}$$

$$x = r\cos\theta$$

$$\sin\theta = \frac{y}{r}$$

$$y = r\sin\theta$$

$$x^2 + y^2 = r^2$$

$$\tan\theta = \frac{y}{x}$$

Convert to Rectangular Coordinates.

$$x = r\cos\theta$$

$$* \quad y = r\sin\theta$$

$$\left(-3, \frac{2\pi}{3}\right)$$

$$x = -3\cos\left(\frac{2\pi}{3}\right)$$

$$= -3\left(-\tfrac{1}{2}\right) = \tfrac{3}{2}$$

$$y = -3\sin\left(2\pi/3\right)$$

$$= -3\left(\sqrt{3}/2\right)$$

$$= -\frac{3\sqrt{3}}{2}$$

$$\boxed{\left(\underset{x}{\frac{3}{2}}, \underset{y}{-\frac{3\sqrt{3}}{2}}\right)}$$

Convert to Rectangular Coordinates.

$$\left(2, \frac{7\pi}{4}\right)$$

$$x = r\cos\theta$$

$$* \quad y = r\sin\theta$$

$$x = 2\cos\left(\frac{7\pi}{4}\right)$$

$$= 2\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}$$

$$y = 2\sin\left(\frac{7\pi}{4}\right)$$

$$= 2\left(-\frac{\sqrt{2}}{2}\right) = -\sqrt{2}$$

$$\boxed{\left(\sqrt{2}, -\sqrt{2}\right)}$$

Convert to Polar Coordinates.

$(-4, 4)$

Q II

$r^2 = (-4)^2 + (4)^2$

$r^2 = 16 + 16$

$r^2 = 32$

$r = \pm\sqrt{32}$

$r = \pm 4\sqrt{2}$

* I usually pick the positive value

$x^2 + y^2 = r^2$

* $\tan\theta = \dfrac{y}{x}$

$\tan\theta = \dfrac{4}{-4}$

$\tan\theta = -1$

$\theta = \tan^{-1}(-1)$

$\theta = 3\pi/4$

$\boxed{(4\sqrt{2}, 3\pi/4)}$

Convert to Polar Coordinates.

$$(-2, -2\sqrt{3})$$

Q III

$$x^2 + y^2 = r^2$$

$$* \quad \tan\theta = \frac{y}{x}$$

$r^2 = (-2)^2 + (-2\sqrt{3})^2$

$r^2 = 4 + 12$

$r^2 = 16$

$r = \pm 4$

$\tan\theta = \frac{-2\sqrt{3}}{-2}$

$\tan\theta = \sqrt{3}$

$\theta = \tan^{-1}(\sqrt{3})$

$\theta = \frac{4\pi}{3}$

$$\boxed{\left(4, \frac{4\pi}{3}\right)}$$

Write the equation using polar coordinates.

$$3xy = 2$$

$$3(r\cos\theta)(r\sin\theta) = 2$$

$$\boxed{3r^2\cos\theta\sin\theta = 2}$$

* $x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

$\tan\theta = \dfrac{y}{x}$

Write the equation using polar coordinates.

$$2x = y^2$$

$$2r\cos\theta = (r\sin\theta)^2$$

$$\boxed{2r\cos\theta = r^2\sin^2\theta}$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$x^2 + y^2 = r^2$$

$$\tan\theta = \frac{y}{x}$$

Write the equation using rectangular coordinates.

$r \cdot 2r = \cos\theta \cdot r$ ← close

$x = r\cos\theta$ *

$y = r\sin\theta$

$x^2 + y^2 = r^2$

$\tan\theta = \frac{y}{x}$

$2r^2 = r\cos\theta$

$\boxed{2(x^2+y^2) = x}$

Write the equation using rectangular coordinates.

$(r)^2 = (7)^2$

$r^2 = 49$

$x^2 + y^2 = 49$

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

$\tan\theta = \dfrac{y}{x}$