

## **Quiz 11.1 Retake**

1. Find the first 3 terms:  $a_1 = -2$ ;  $a_n = 3a_{n-1} - 4$ .
2. Find the sum.

$$\sum_{k=1}^6 (-1)^k k!$$

## Lesson 11.3: Geometric Sequences and Series

In a geometric sequence, the ratio between successive terms is constant.

$r$

<b>Recursive Definition</b>	$a_1 = a$ $a_n = r \cdot a_{n-1}$
<b>nth Term Definition</b>	$\{a \cdot r^{n-1}\}$

$$a_1 = a$$

To find  $r$  divide two successive terms:  $r = \frac{a_n}{a_{n-1}}$

$$a_2 = ar$$

$$a_3 = ar \cdot r = ar^2$$

$$a_4 = ar^3$$

$$\vdots$$
$$a_n = ar^{n-1}$$

Which of the following are geometric?

$$2, 6, 18, 54, \dots$$

$$\{2n - 5\}$$

Yes. Geometric

No. Arithmetic

$$r = 3$$

$$\{3n^2 + 1\}$$

$$\{2(4)^n\}$$

Not Geometric

Geometric

$$r = 4$$

Suppose  $a_1 = -2$  and  $r = 3$ , then find the following:

a) nth term  $a \cdot r^{n-1}$

b) 10<sup>th</sup> term

c) Recursive Definition

(a) nth term

$$\{-2(3)^{n-1}\}$$

(c) Recursive

$$a_1 = -2$$

$$a_n = 3a_{n-1}$$

(b) 10<sup>th</sup> term

$$-2(3)^{10-1} = \boxed{-39,364}$$

Suppose  $a_1 = 10$  and  $r = \frac{1}{2}$ , then find the following:

- a) nth term
- b) 5<sup>th</sup> term
- c) Recursive Definition

(a)  $\left\{ 10\left(\frac{1}{2}\right)^{n-1} \right\}$

(c)  $a_1 = 10$   
 $a_n = \frac{1}{2} a_{n-1}$

(b)  $10\left(\frac{1}{2}\right)^{5-1} = 0.625$   
or  
 $5/8$

Find the 9<sup>th</sup> term in the given geometric sequence  $\underline{10}, \underline{9}, \underline{\frac{81}{10}}, \underline{\frac{729}{100}}, \dots$

$$r = \frac{9}{10} = .9$$

$$\left\{ 10(.9)^{n-1} \right\}$$

$$10(.9)^{9-1} \approx \boxed{4.\overline{305}}$$

## Finding the Sum of Finite Geometric Series

$$S_n = \frac{a_1 \cdot (1 - r^n)}{1 - r} \quad *$$

$$\frac{\underline{a_1} - a_1 \cdot r^n}{1 - r}$$

$$\frac{a_1 - r \cdot \underline{a_1 \cdot r^{n-1}}}{1 - r}$$

$$S_n = \frac{a_1 - r \cdot a_n}{1 - r}$$

## Finding the Sum of Infinite Geometric Series

If  $|r| < 1$  then,

$$S = \frac{a_1}{1 - r}$$

$$r^3 = r \cdot r^2$$

Find the sum of  $a_1 + a_2 + a_3 + \dots + a_n$

4)  $S_n = \frac{a_1 \cdot (1 - r^n)}{1 - r}$

4)  $S_n = \frac{a_1 - r \cdot a_n}{1 - r}$

$$S_n = \frac{6 - 2 \cdot 3072}{1 - 2} = 6,138$$

$$S = \frac{a_1}{1 - r}$$

Top Formula

$$6(2)^{n-1} = 3072$$

$$n = 10$$

$$2^{n-1} = 512$$

$$2^{n-1} = 2^9$$

$$n-1 = 9$$

Write the following in summation notation:

$$6 + 12 + 24 + \cdots + 3072$$

$$\sum_{k=1}^{10} 6(2)^{k-1}$$

*number of terms* ←

Find the sum.

$$1600 + 800 + 400 + \dots + 6.25$$

$r = \frac{1}{2}$

$$S_n = \frac{(1600 - \frac{1}{2}(6.25))}{(1 - \frac{1}{2})}$$
$$\approx \boxed{3,193.75}$$

$$S_n = \frac{a_1 \cdot (1 - r^n)}{1 - r}$$

\*

$$S_n = \frac{a_1 - r \cdot a_n}{1 - r}$$

$$S = \frac{a_1}{1 - r}$$

Find the sum.

$$2 + \underbrace{\frac{4}{3} + \frac{8}{9}}_{\text{red underline}} + \dots$$

$$r = \frac{4/3}{2} = \frac{2}{3}$$

$$S_n = \frac{a_1 \cdot (1 - r^n)}{1 - r}$$

$$S_n = \frac{a_1 - r \cdot a_n}{1 - r}$$

~~$$S = \frac{a_1}{1 - r}$$~~

$$S = \frac{2}{1 - \frac{2}{3}} = \boxed{6}$$

Find the sum.

$$n = 10$$
$$\sum_{k=1}^{10} 2(3)^{k-1}$$

$$S_n = \frac{2(1-3^{10})}{1-3} = \boxed{59,048}$$

~~$$S_n = \frac{a_1 \cdot (1 - r^n)}{1 - r}$$~~

$$S_n = \frac{a_1 - r \cdot a_n}{1 - r}$$

$$S = \frac{a_1}{1 - r}$$

Find the sum.

$$\sum_{k=1}^{\infty} 3 \left(\frac{1}{4}\right)^{k-1}$$

$$S = \frac{3}{1 - \frac{1}{4}} = \boxed{4}$$

$$S_n = \frac{a_1 \cdot (1 - r^n)}{1 - r}$$

$$S_n = \frac{a_1 - r \cdot a_n}{1 - r}$$

~~$$S = \frac{a_1}{1 - r}$$~~

Find the sum.

$$\sum_{k=1}^{\infty} 2(3)^{k-1}$$



r is bigger  
than 1

$$S_n = \frac{a_1 \cdot (1 - r^n)}{1 - r}$$

$$S_n = \frac{a_1 - r \cdot a_n}{1 - r}$$

$$S = \frac{a_1}{1 - r}$$

Diverges