Lesson 11.2: Arithmetic Sequences and Series

In an arithmetic sequence, the difference between successive terms is constant (or the same number).

$a_{n-1} + 7$

Recursive Definition

$$a_1 = a$$

$$a_n = a_{n-1} + d$$

* *d* represents the common difference
* *a* represents the first term

Note: When writing a recursive definition, the first term **must** be stated.

Recursive Definition

$$a_1 = a$$

$$a_n = a_{n-1} + d$$

$a_1 = a$

$a_2 = a + d$

$a_3 = a + 2d$

$a_4 = a + 3d$

$\vdots$

$a_{10} = a + 9d$

$\vdots$

$a_n = a + (n-1)d$

nth Term Definition
(also called the explicit definition)

$\{a + d(n-1)\}$

$a + dn - d$

$dn + (a - d)$

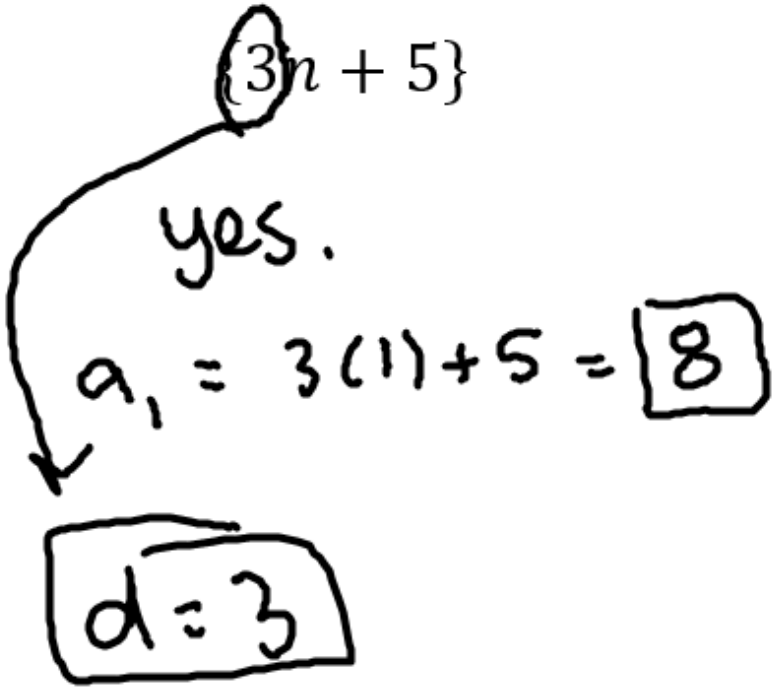
Is the following sequence arithmetic? If yes, find the first term and common difference.

$\{n^2 - 2\}$

↓

not arithmetic

Is the following sequence arithmetic? If yes, find the first term and common difference.



Note: To find the first term, **always** plug in 1 for *n*.

Find the common difference and write the first four terms of $\left\{5 - \frac{1}{2}n\right\}$

$d = -\frac{1}{2}$

$a_1 = 5 - \frac{1}{2}(1) = 4.5$

$a_2 = 4$

$a_3 = 3.5$

$a_4 = 3$

Suppose $a_1 = 4$ and $d = -7$, find the following:

a) nth term
b) 10^{th} term
c) Recursive definition.

(a) nth term → $a + (n-1)d$

$dn + (a-d)$

$-7n + (4 - -7)$

$\{-7n + 11\}$

(b) 10th Term:

$-7(10) + 11 = -59$

(c) Recursive

$a_1 = 4$

$a_n = a_{n-1} - 7$

Suppose $a_1 = 2$ and $d = 4$, find the following:

a) nth term
b) 14^{th} term
c) Recursive definition.

(a) n^{th} term : $dn + (a - d)$

$\{4n - 2\}$

(b) 14th term

$a_{14} = 4(14) - 2 = \boxed{54}$

(c) Recursive

$a_1 = 2$

$a_n = a_{n-1} + 4$

Find the 99^{th} term in the given arithmetic sequence 4, 7, 10,...

$a_1 = 4$

$d = 3$

$dn + (a - d)$

nth term: $\{3n + 1\}$

99th: $3(99) + 1 = \boxed{298}$

Find the following:

$a_9 = 8$; $a_{17} = 32$

a) 1st term
b) Common difference
c) Recursive definition
d) nth term definition : $dn + (a - d)$

Common difference

$$d = \frac{32 - 8}{17 - 9} = 3$$

First Term:

$8 = 3(9) + a - 3$

$a = -16$

Nth term

$3n - 19$

Recursive

$a_1 = -16$

$a_n = a_{n-1} + 3$

Finding the Sum of an Arithmetic Series

$$S_n = \frac{n}{2}(a_1 + a_n)$$

n: # of terms

a_1: 1st term

a_n: Last term

Find the sum of $8 + 11 + 14 + \cdots + 68$

$d = +3 \quad +3$

1.

2.

3.

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_n = \frac{21}{2}(8 + 68) = \boxed{798}$$

nth term:

$\{3n + 5\}$ ← Last term

$3n + 5 = 68$

$3n = 63$

$n = 21$

Write the following in summation notation $8 + 11 + 14 + \cdots + 68$

$$\sum_{k=1}^{n} (Explicit\ Formula)$$

$$\sum_{k=1}^{21} 3k+5$$

Find the sum of $-1 + 3 + 7 + \cdots + 151$

+4

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$= \frac{39}{2}(-1 + 151) = 2925$$

Find n:

$$151 = 4n - 5$$

$$156 = 4n$$

$$n = 39$$

Write the following in summation notation $-1 + 3 + 7 + \cdots + 151$

$$\sum_{k=1}^{n} (Explicit\ Formula)$$

$$\sum_{K=1}^{39} 4K-5$$

The corner section of a football stadium has 15 seats in the first row and 40 rows in all. Each successive row contains two additional seats. How many seats are in this section?

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$a_1 = 15$

$n = 40$

$d = 2$

$a_n = 2n + 13$

$a_{40} = 2(40) + 13 = 93$

$\frac{40}{2}(15 + 93)$

$= 2{,}160$ Seats