Lesson 11.1: Sequences and Series

Sequence: a pattern of numbers whose domain is the set of positive integers (i.e. 1, 2, 3,...)

Example: 2, 7, 12, 17,...

***nth* Term Definition:** a rule that produces the *nth* term when plugging *n* into the formula. (explicit definition)

Example: $\{5n - 9\}$

Recursive Definition: a rule that uses the preceding term to define the next term of the sequence. The definition must have two parts: (1) the first term and (2) the recursive formula.

Example: $a_1 = -3$ ← 1st term

$$\underbrace{a_n}_{\text{nth term}} = 2\underbrace{a_{n-1}}_{\text{n-1 term or previous term}} + 7$$

Write the first 5 terms of the following sequence.

$a_n = \{2n^2 - 9\}$ (nth term definition)

$a_1 = 2(1)^2 - 9 = -7$

$a_2 = 2(2)^2 - 9 = -1$

$a_3 = 2(3)^2 - 9 = 9$

$a_4 = 2(4)^2 - 9 = 23$

$a_5 = 2(5)^2 - 9 = 41$

$\boxed{-7, -1, 9, 23, 41}$

Write the first 5 terms of the following sequence.

$$\left\{\frac{n!}{n+2}\right\}$$

nth term definition

$$a_1 = \frac{1!}{1+2} = \frac{1}{3}$$

$$a_2 = \frac{2!}{2+2} = \frac{2}{4} = \frac{1}{2}$$

$$a_3 = \frac{3!}{3+2} = \frac{3\cdot 2\cdot 1}{3+2} = \frac{6}{5}$$

$$a_4 = \frac{4!}{4+2} = \frac{24}{6} = 4$$

$$a_5 = \frac{5!}{5+2} = \frac{120}{7}$$

$$\boxed{\frac{1}{3}, \frac{1}{2}, \frac{6}{5}, 4, \frac{120}{7}}$$

Write the first 5 terms of the following sequence.

$$a_1 = 2$$

* Recursive

$$a_n = 4\boxed{a_{n-1}} - 4$$

previous term

$a_1 = 2$

$a_2 = 4(2) - 4 = 4(2) - 4 = 4$

$a_3 = 4(4) - 4 = 12$

$a_4 = 4(12) - 4 = 44$

$a_5 = 4(44) - 4 = 172$

$$\boxed{2, 4, 12, 44, 172}$$

Write the first 5 terms of the following sequence.

$$a_1 = 2$$

Recursive

$$a_2 = -3$$

$$a_n = 2\boxed{a_{n-1}} + \boxed{a_{n-2}}$$ ← two terms ago

previous term

$a_1 = 2$

$a_2 = -3$

$a_3 = 2(-3) + 2 = -4$

$a_4 = 2(-4) + (-3) = -11$

$a_5 = 2(-11) + -4 = -26$

$$\boxed{2, -3, -4, -11, -26}$$

Summation Notation

Plug in to K

ending term

add up values

$$\sum_{k=1}^{4} \underline{2k-3}$$

nth term of sequence

first term

$a_1 = 2(1) - 3 = -1$

$a_2 = 2(2) - 3 = 1$

$a_3 = 2(3) - 3 = 3$

$a_4 = 2(4) - 3 = 5$

$= -1 + 1 + 3 + 5 = \boxed{8}$

Find the sum.

$$\sum_{k=1}^{7} k^2 + 5 = 6 + 9 + 14 + 21 + 30 + 41 + 54$$

$$= \boxed{175}$$

$a_1 = 1^2 + 5 = 6$

$a_2 = 2^2 + 5 = 9$

$a_3 = 3^2 + 5 = 14$

$a_4 = 4^2 + 5 = 21$

$a_5 = 5^2 + 5 = 30$

$a_6 = 6^2 + 5 = 41$

$a_7 = 7^2 + 5 = 54$

Find the sum.

$$\sum_{k=1}^{5}(-1)^k(k+1)! = -2 + 6 + -24 + 120 + -720$$

$$= \boxed{-620}$$

$a_1 = (-1)^1(1+1)! = -2$

$a_2 = (-1)^2(2+1)! = 6$

$a_3 = (-1)^3(3+1)! = -24$

$a_4 = (-1)^4(4+1)! = 120$

$a_5 = (-1)^5(5+1)!$

$= -720$

Write the following in summation notation.

$$\frac{1}{6}+\frac{2}{7}+\frac{3}{8}+\cdots+\frac{17}{17+5}$$

$$\boxed{\sum_{k=1}^{17} \frac{k}{k+5}}$$

Write the following in summation notation.

$$-2 + 3 + 8 + 13 + \cdots + [5(23) - 7]$$

$$\boxed{\sum_{K=1}^{23} 5K - 7}$$

$5(1) - 7 = -2$ ✓

Write the following in summation notation.

$$\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}+\cdots+(-1)^{13-1}\left(\frac{1}{2}\right)^{13}$$

$$\boxed{\sum_{K=1}^{13}(-1)^{K-1}\left(\frac{1}{2}\right)^{K}}$$