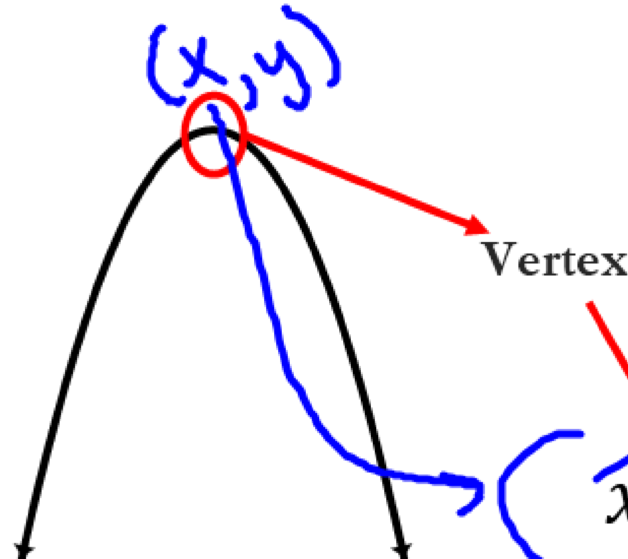
Lesson 10.2: Design Problems

Maximizing/Minimizing Data:

- Visualize and write equations based on the given information
- Determine which equation needs to be minimized or maximized
- Substitute if necessary so that the max/min equation has one variable
- Find the max or min (if the equation is not a quadratic, use a graphing utility)

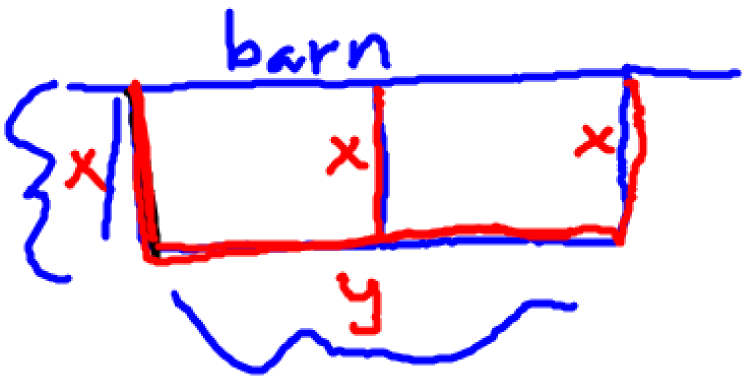
desmos.com

If a quadratic equation:



$$x = -\frac{b}{2a}$$

A farmer has enough fencing to build 90 feet of fence. He wishes to build two adjacent rectangular pens next to his barn, with the barn wall forming one side of each pen. What dimensions should he make the pens so as to enclose the maximum area?



$A = xy$

$3x + y = 90 \rightarrow y = 90 - 3x$

$-3x \qquad -3x$

$\text{Area} = x(90 - 3x)$

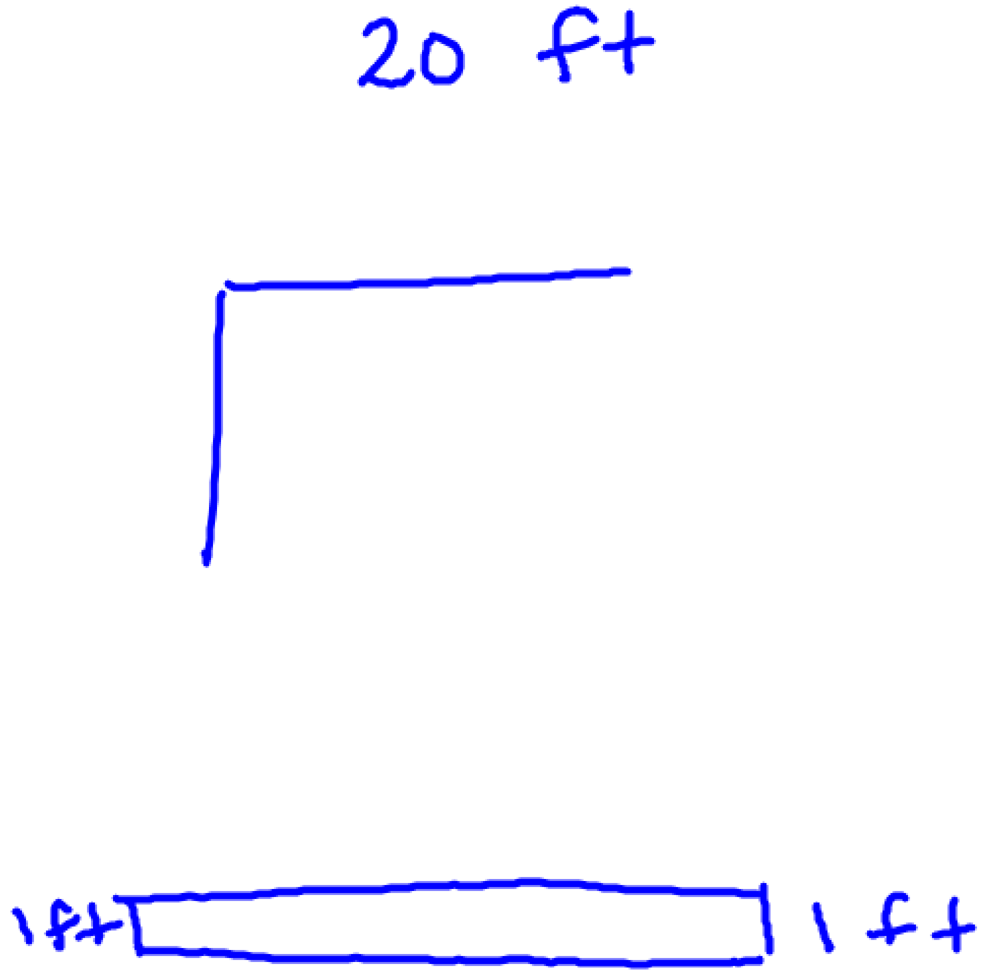
$= 90x - 3x^2$

$= -3x^2 + 90x$

$x = \frac{-b}{2a}$

$x = \frac{-90}{2(-3)} = 15, \quad y = 90 - 3(15) = 45$

$\boxed{\text{15 ft by 45 ft}}$


20 ft
1 ft
1 ft

A baseball stadium normally can sell 2,000 hot dogs at a game if they charge \$2.50 each. They also notice that if they raise the price by \$0.25, they sell 100 fewer hotdogs. Determine the price they should charge to maximize the revenue.

x - # of increases of the price

Revenue=*(Quantity)(Price)*

Quantity $= 2000 - 100x$

Price $= 2.50 + 0.25x$

$$(2000 - 100x)(2.5 + .25x)$$

$$-25x^2 + 250x + 5000$$

$$x = -\frac{b}{2a} = \frac{-250}{2(-25)} = \boxed{5}$$

Price: $2.5 + 0.25(5) = \$3.75$

A designer wants to manufacture storage bins ~~in the shape of~~ rectangular prisms. Each bin will have a volume of 10 cubic feet. The designer wants to choose the dimensions of a bin so that the length is 2 times the width and so that the surface area is minimized. What dimensions should the company choose for the bin?

$$V = 10 = 2w \cdot w \cdot h$$

$$10 = 2w^2 h \rightarrow h = \frac{10}{2w^2} = \frac{5}{w^2}$$

$$SA = 2 \cdot 2w^2 + 2 \cdot 2w \cdot h + 2 \cdot wh$$

$$= 4w^2 + 6wh$$

$$= 4w^2 + 6w\left(\frac{5}{w^2}\right)$$

Desmos:

$w = 1.554$ ft

$l = 1.554 \cdot 2 = 3.08$ ft

$h = \frac{5}{1.554^2} = 2.07$ ft

A company manufactures cans that have a volume of 1500 cubic centimeters. The company wants to minimize the cost of the materials by minimizing the amount of aluminum used to produce the cans. What dimensions should you choose for the cans?

Vol: * $1500 = \pi r^2 h$

$$h = \frac{1500}{\pi r^2}$$

SA: $2\pi r^2 + 2\pi r h$

* $2\pi r^2 + 2\pi r\left(\frac{1500}{\pi r^2}\right)$

<u>Desmos:</u>

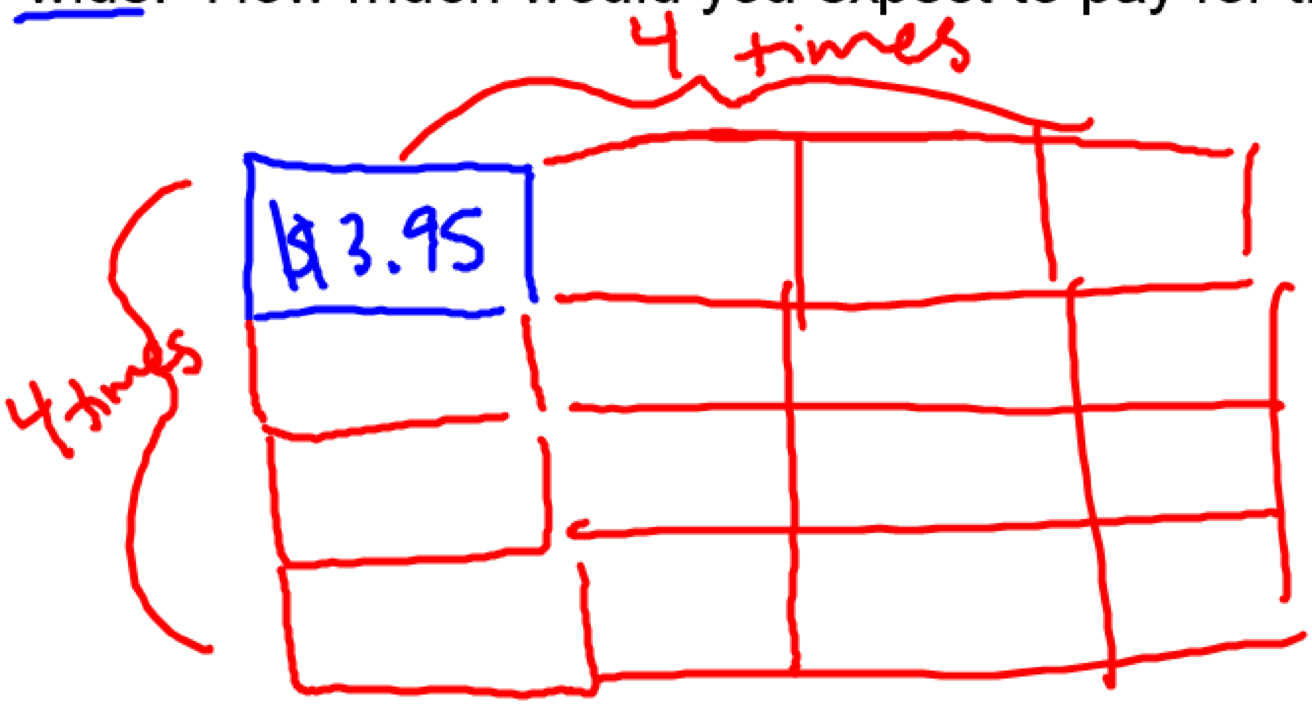
$r = 6.204$ cm

$$h = \frac{1500}{\pi(6.204)^2} \approx 12.405 \text{ cm}$$

<u>Size Changes</u>

- Visualize the change in size for each dimension
- Multiply by the appropriate amount.

An embroidered placemat costs \$3.95. An embroidered tablecloth is similar to the placemat, but four times as long and four times as wide. How much would you expect to pay for the tablecloth?



$\$3.95\underset{L}{(4)}\underset{W}{(4)}$

$\boxed{\$63.20}$

A small box holds eight chocolates. A larger box doubles the dimensions. How many chocolates will the box hold?

$$\underset{}{8}\underset{L}{(2)}\underset{W}{(2)}\underset{H}{(2)} = 64$$

Chocolates