Lesson 1.6: Modeling with Piecewise and Composite Functions

Honors Math 3

Creating a Model of a Piecewise Function

1. Identify the intervals where the model differs.
2. Determine an equation for each interval.
3. Combine equations and intervals to create the piecewise function

48) In May 2003, Nicor Gas had the following rate schedule for natural gas usage:

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- Monthly Customer Charge:	$6.45
- Distribution Charge	
1st 20 therms	$0.2012/therm
Next 30 therms	$0.1117/therm
Over 50 therms	$0.0374/therm
-Gas Supply charge	$0.7268/therm

a) What is the charge for using 40 therms in a month?

b) What is the charge for using 202 therms in a month?

c) Construct a function that gives the monthly charge *C* for *x* therms of gas.

d) Graph this function.

a) $\underbrace{6.45}_{\text{Monthly}} + \underbrace{0.2012(20) + 0.1117(20)}_{\text{Distribution}} + \underbrace{0.7268(40)}_{\text{gas}}$

$= \boxed{\$41.78}$

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Over 50 therms	$0.0374/therm
-Gas Supply charge	$0.7268/therm

b) What is the charge for using 202 therms in a month?

b) $\underbrace{6.45}_{\text{Montly Charge}} + \underbrace{0.2012(20) + 0.1117(30) + 0.0374(152)}_{\text{Distribution Charge}} + \underbrace{0.7268(202)}_{\text{Gas Charge}}$

$= \boxed{\$166.32}$

48) In May 2003, Nicor Gas had the following rate schedule for natural gas usage:

Page 116

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-Gas Supply charge	$0.7268/therm

c) Construct a function that gives the monthly charge *C* for *x* therms of gas.

- $0 \le x \le 20$

  $6.45 + 0.2012x + 0.7268x$

- $20 < x \le 50$

  $6.45 + 0.2012(20) + 0.1117(x-20) + 0.7268x$

- $x > 50$

  $6.45 + 0.2012(20) + 0.1117(30) + 0.7268(x-50) + 0.7268x$

$$\boxed{C(x) = \begin{cases} 0.928x + 6.45 & 0 \le x \le 20 \\ 0.8385x + 8.24 & 20 < x \le 50 \\ 0.7642x + 11.955 & x > 50 \end{cases}}$$

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d) Graph this function.

53) Holders of credit cards issued by banks, department stores, oil companies, and so on, receive bills each month that state minimum amounts that must be paid by a certain due date. The minimum due depends on the total amount owed. One such credit card company uses the following rules: For a bill of less than \$10, the entire amount is due. For a bill of at least \$10 but less than \$500, the minimum due is \$10. A minimum of \$30 is due on a bill of at least \$500 but less than \$1000, a minimum of \$50 is due on a bill of at least \$1000 but less than \$1500, and a minimum of \$70 is due on bills of \$1500 or more. Find the function *f* that describes the minimum payment due on a bill of *x* dollars. Graph *f*.

$$f(x) = \begin{cases} x & 0 \le x < 10 \\ 10 & 10 \le x < 500 \\ 30 & 500 \le x < 1000 \\ 50 & 1000 \le x < 1500 \\ 70 & x \ge 1500 \end{cases}$$

53) Holders of credit cards issued by banks, department stores, oil companies, and so on, receive bills each month that state minimum amounts that must be paid by a certain due date. The minimum due depends on the total amount owed. One such credit card company uses the following rules: For a bill of less than \$10, the entire amount is due. For a bill of at least \$10 but less than \$500, the minimum due is \$10. A minimum of \$30 is due on a bill of at least \$500 but less than \$1000, a minimum of \$50 is due on a bill of at least \$1000 but less than \$1500, and a minimum of \$70 is due on bills of \$1500 or more. Find the function *f* that describes the minimum payment due on a bill of *x* dollars. Graph *f*.

Keys to Creating a Model with Composite Functions

1. Identify the 3 variables and 2 functions given.
2. Identify the common variable in both functions

$\boxed{\text{#64 on Page 255}}$

The volume V (in cubic meters) of a hot-air balloon is given by $V(r) = \frac{4}{3}\pi r^3$ where r is the radius of the balloon (in meters). If the radius r is increasing with time t (in seconds) according to the formula $r(t) = \frac{2}{3}t^3$, $t \geq 0$, find the volume V as a function of time t.

$$V(r) = \frac{4}{3}\pi r^3$$

$$r(t) = \boxed{\frac{2}{3}t^3}$$

$$V(r(t)) = \frac{4}{3}\pi\left(\frac{2}{3}t^3\right)^3$$

$$= \frac{4}{3}\pi \cdot \frac{8}{27}t^9 = \boxed{\frac{32}{81}\pi t^9}$$

$$V(t) = \frac{32}{81}\pi t^9$$

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The price *p*, in dollars, of a certain commodity and the quantity *x* sold obey the demand equation

$$p = -\frac{1}{5}x + 200, \qquad 0 \le x \le 1000$$

Suppose that the cost *C*, in dollars, of producing *x* units is

$$C = \frac{\sqrt{x}}{10} + 400$$

Assuming that all items produced are sold, find the cost *C* as a function of the price *p*. → we want $C(p)$. So, we need to solve the top equation for x.

$$p = -\frac{1}{5}x + 200$$
$$-200 \qquad\qquad -200$$

$$p - 200 = -\frac{1}{5}x \rightarrow x = -5p + 1000$$

$$C(x) = \frac{\sqrt{x}}{10} + 400$$

$$C(p) = \frac{\sqrt{-5p+1000}}{10} + 400$$

Review!!!!

Domain

Average Rate of Change

Graphical Analysis

Graphing Parent Functions

Piecewise Functions

Graphing Unique Parent Functions

Domain

Find the domain of the following functions.

$$y = \frac{4}{x+2}$$

$x+2 \neq 0$

$\boxed{x \neq -2}$

$$y = \sqrt{1-2x}$$

$1-2x \geq 0$

$\boxed{\tfrac{1}{2} \geq x}$

$$y = x^2 - 5x + 4$$

No division or square roots.

$\boxed{\mathbb{R}}$

$$y = \frac{x+5}{\sqrt{3-x}}$$

$3-x > 0$

$\boxed{3 > x}$

Average Rate of Change

For the following, use the function $f(x) = x^2 - 5x + 6$.

a) Find the average rate of change from 0 to x.

x	y
0	6
x	x^2-5x+6

$$\frac{(x^2-5x+6)-6}{x-0} = \frac{x^2-5x}{x} = \frac{x(x-5)}{x} = \boxed{x-5}$$

b) Find the average rate of change from 0 to 7.

x

from a) $x-5$

$7-5=\boxed{2}$

c) Find the equation of the secant line containing (0, f(0)) & (7, f(7)).

$m=2$ from a we have the point $(0,6)$ which means 6 is the y-intercept.

$$\boxed{y=2x+6}$$

Graphical Analysis

Domain: $(-\infty, \infty)$

Range: $(-\infty, 2]$

x-intercept(s): $(0,0)$ + $(2.5, 0)$

y-intercept: $(0,0)$

Increasing: $(0,1)$

Decreasing: $(-1,0)$ + $(1,\infty)$

Constant: $(-\infty, -1)$

Positive: $(-\infty, 0)$ + $(0, 2.5)$

Negative: $(2.5, \infty)$

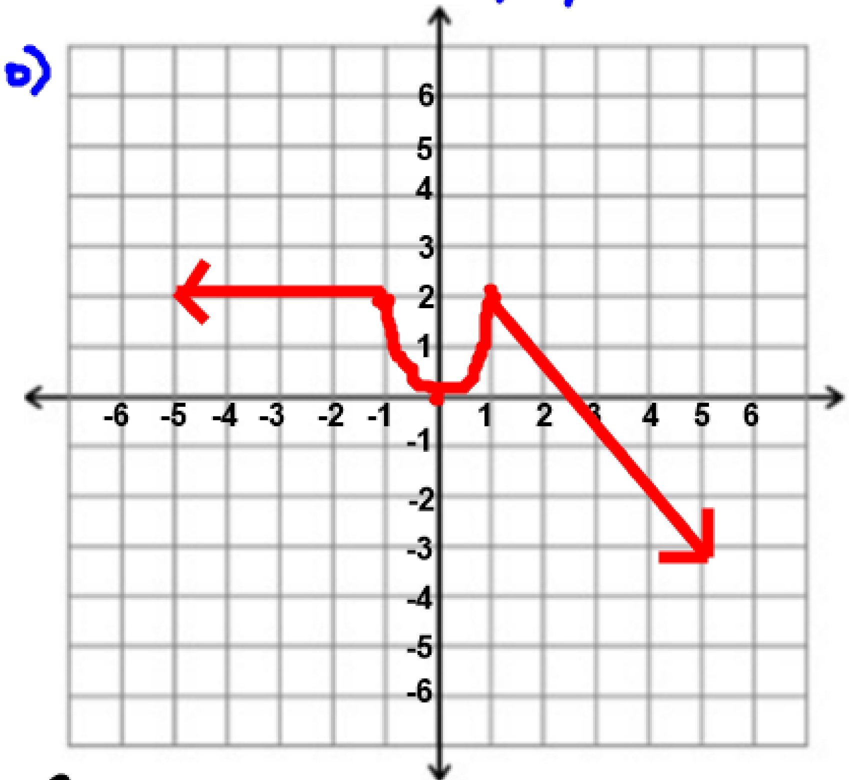
End Behavior:

L→2: As $x \to -\infty$, $y \to 2$

R↓: As $x \to \infty$, $y \to -\infty$

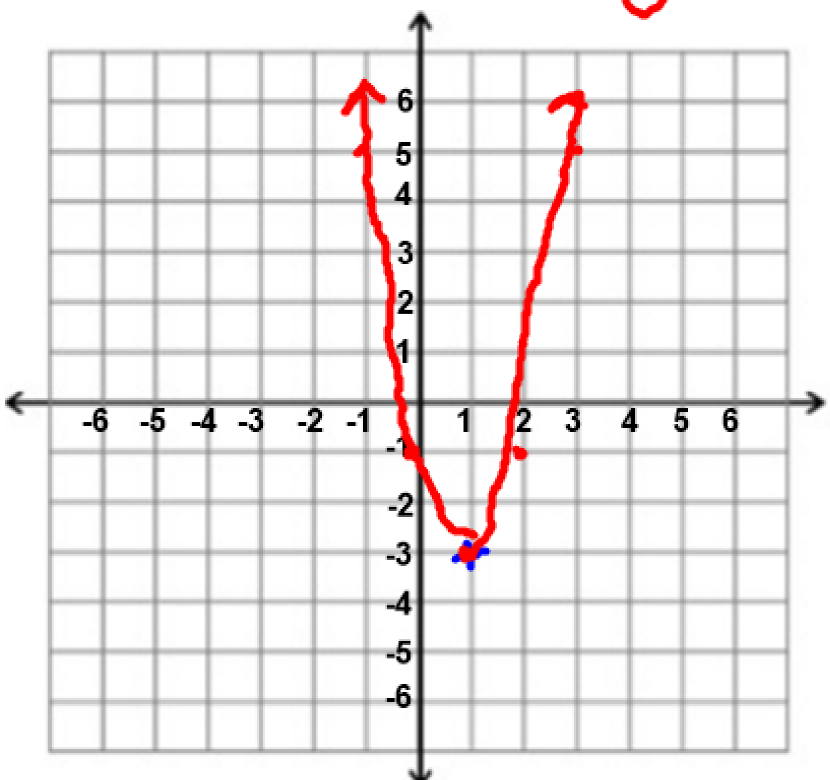
Local Max: $(1,2)$

Local Min: $(0,0)$



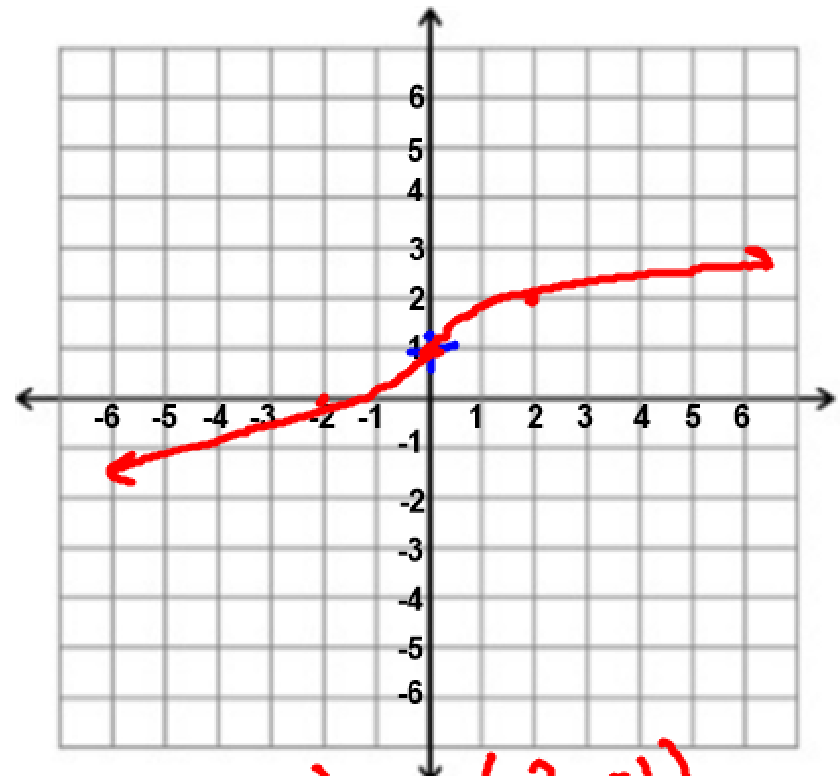
Graphing Parent Functions

$$f(x) = 2(x-1)^2 - 3$$



$(-1,1) \rightarrow (-1,2)$
$(0,0) \rightarrow (0,0)$
$(1,1) \rightarrow (1,2)$

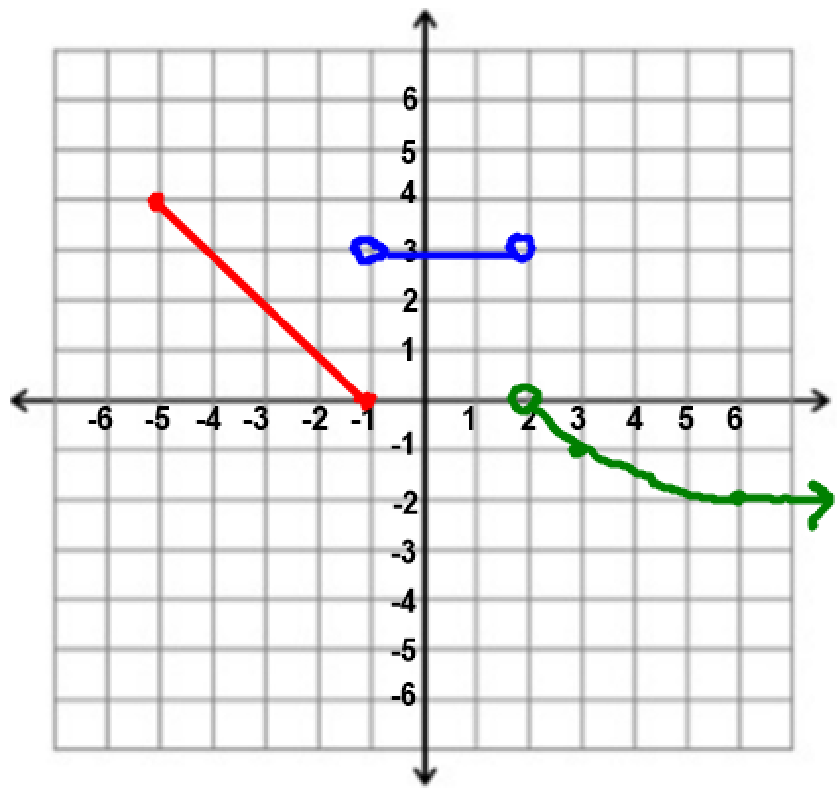
$$g(x) = \sqrt[3]{\frac{x}{2}} + 1$$



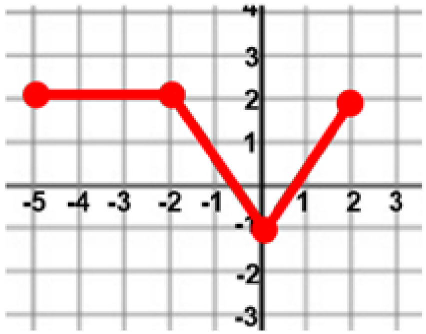
$(-1,-1) \rightarrow (-2,-1)$
$(0,0) \rightarrow (0,0)$
$(1,1) \rightarrow (2,1)$

Piecewise Functions

$$f(x) = \begin{cases} -x-1 & -5 \le x \le -1 \\ 3 & -1 < x < 2 \\ -\sqrt{x-2} & x > 2 \end{cases}$$



Graphing Unique Parent Functions

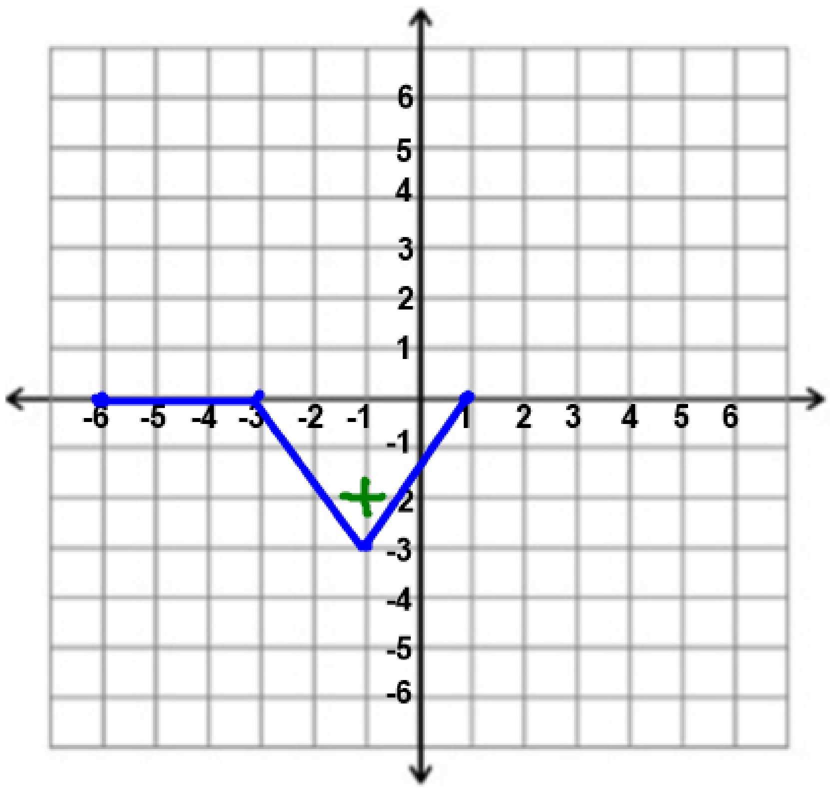


Key Pts:

(-5, 2) (-2, 2) (0, -1) (2, 2)

(-5, 2) → (-2.5, 2)
(-2, 2) → (-1, 2)
(0, -1) → (0, -1)
(2, 2) → (1, 2)

$f(x+1)-2$



$f(2x)$