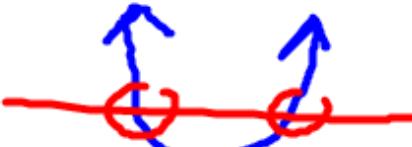
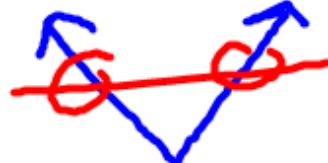


Lesson 1.5: Inverse Functions

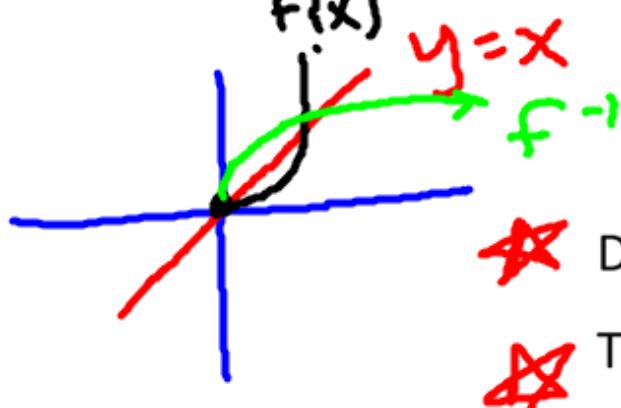
- * **One to One Function**: A function where every y-value has one and only one x-value. (One to One functions pass both the vertical and horizontal line test).


One - to - one

 No

 No

* Inverse Function: $f(f^{-1}(x)) = x = f^{-1}(f(x))$



★ Domain of $f(x)$ is the Range of $f^{-1}(x)$

★ The Range of $f(x)$ is the Domain of $f^{-1}(x)$

Verify or refute whether $f(x)$ and $g(x)$ are inverses.

* Ex: $f(x) = 2x + 9$

$$g(x) = \frac{x}{2} - \frac{9}{2}$$

$$f(g(x)) = 2\left(\frac{x}{2} - \frac{9}{2}\right) + 9$$

$$= x - 9 + 9$$

$$= x \checkmark$$

$$g(f(x)) = \frac{2x+9}{2} - \frac{9}{2}$$

$$= \frac{2x}{2} + \frac{9}{2} - \frac{9}{2}$$

$$= x \checkmark$$

Inverses ✓

$$g(f(x)) = x$$

$$f(g(x)) = x$$

* Ex: $f(x) = x^3 + 1$

$$g(x) = \sqrt[3]{x} - 1$$

$$f(g(x)) = (\sqrt[3]{x} - 1)^3 + 1$$

$$(\sqrt[3]{x} - 1)(\sqrt[3]{x} - 1)(\sqrt[3]{x} - 1)$$

$$= x - 3(\sqrt[3]{x})^2 + 3(\sqrt[3]{x}) - 1 + 1$$

⇒ Not inverses

Find the inverse. Then, find the domain and range of f and f^{-1} .

$$f(x) = 5x - 7$$

$$y = 5x - 7$$

	f	f^{-1}
Domain	\mathbb{R}	\mathbb{R}
Range	\mathbb{R}	\mathbb{R}

Solve for x :

$$\frac{y+7}{5} = \frac{5x}{5}$$

$$x = \frac{y+7}{5} \quad \text{or} \quad \frac{y}{5} + \frac{7}{5}$$

$$f^{-1}(y) = \frac{y+7}{5}$$

Find the inverse. Then, find the domain and range of f and f^{-1} .

$$f(x) = 2x^2 - 3, \quad x \leq 0$$

$$y = 2x^2 - 3$$

$$y + 3 = 2x^2$$

$$\pm \sqrt{\frac{y+3}{2}} = \sqrt{x^2}$$

$$x = -\sqrt{\frac{y+3}{2}}$$

$$f^{-1}(y) = -\sqrt{\frac{y+3}{2}}$$

	f	f^{-1}
Domain	$x \leq 0$	$y \geq -3$
Range	$y \geq -3$	$x \leq 0$

$$\frac{y+3}{2} \geq 0$$

$$y+3 \geq 0$$

$$y \geq -3$$

Find the inverse. Then, find the domain and range of f and f^{-1} .

$$f(x) = \frac{3}{x+1}$$

$$(x+1) \cdot y = \frac{3}{x+1} \cdot (x+1)$$

$$\frac{(x+1)y}{y} = \frac{3}{y}$$

$$x+1 = \frac{3}{y} \quad | -1$$

$$x = \frac{3}{y} - 1$$

	f	f^{-1}
Domain	$x \neq -1$	$y \neq 0$
Range	$y \neq 0$	$x \neq -1$

$$f^{-1}(y) = \frac{3}{y} - 1$$

Find the inverse. Then, find the domain and range of f and f^{-1} .

$$f(x) = \frac{x}{4-x}$$

$$(4-x) \cdot y = \frac{x}{4-x} \cdot (4-x)$$

$$\cancel{(4-x)}y = x$$

$$4y - xy = x$$
$$+xy \qquad +xy$$

$$4y = \underline{x} + \underline{xy}$$

$$\frac{4y}{1+y} = \frac{x(1+y)}{1+y}$$

	f	f^{-1}
Domain	$x \neq 4$	$y \neq -1$
Range	$y \neq -1$	$x \neq 4$

$$x = \frac{4y}{1+y}$$

$$f^{-1}(y) = \frac{4y}{1+y}$$

Find the inverse.

$$f(x) = 3\sqrt{x+2}$$

$$\frac{y}{3} = \frac{3\sqrt{x+2}}{3}$$

$$\frac{y^2}{9} - 2 = x$$

$$\left(\frac{y}{3}\right)^2 = (\sqrt{x+2})^2$$

$$f^{-1}(y) = \frac{y^2}{9} - 2$$

$$\frac{y^2}{9} = x + 2$$

-2

Find the inverse.

$$f(x) = \frac{1}{(x-2)^2},$$

↓
 $x \geq 2$

$$(x-2)^2 \cdot y = \frac{1}{(x-2)^2} \cdot (x-2)^2$$

$$\frac{(x-2)^2 \cdot y}{(x-2)^2} = \frac{1}{y}$$
$$\sqrt{(x-2)^2} = \sqrt{\frac{1}{y}}$$

$$x-2 = \sqrt{\frac{1}{y}}$$

$$+2 \quad +2$$

$$f^{-1}(y) = \sqrt{\frac{1}{y}} + 2$$